

ACOUSTIC METHODS OF WORK

IN RELATION TO

SYSTEMATIC COMPARATIVE MUSICOLOGY

INCLUDING SOME ACOUSTIC TABLES

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In Memoriam

FREDERIK LASSEN LANDORPH

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PREFACE

The aim of this treatise is:

1. To propose a common international terminology of music; thus, for the white keys of the piano the names	{ c bis	d fes	e eis	f fis	g gis	a ais	b bes	...
and for the black keys	{ cis des	dis es	fis ges	gis as	ais bes	...	

and for double $\sharp\sharp$ and $\flat\flat$ the terminations isis and eses; for syntonic »Comma-tones« the signs $\rightarrow+$ and $\rightarrow-$ placed after the name of the tone to indicate increase or diminution of a Comma = $\frac{81}{80}$; as, for instance, in Indian music: a = 884 cents and a $\rightarrow+$ = 906 cents, or 22 cents more.

Only Ellis's 1200 cents are used (never 1000 millioctaves, or 100 Octav-Zentimeter or 600 centitones), as the number 1200 can be divided evenly by 3, 4×4 and 5×5 and their multiples (as 720 particles can be divided on a string by 3×3 and 4×4 and

Below are given some further proposals concerning terminology; for instance, expression »triple chord« is used for »triad« in opposition to the »Greek Triad« neighbour-tones: des \rightarrow , des and d \rightarrow , and »quadruple chord« for »chord of Seventh«; »Tonics« or starting-point for »key-note«; »Phrygian-Doric« for the Aeolian scale-type, and »Lydian-Phrygian« for the Ionic or Iastic scale-type, according to succession of Tetrachords, Scheme 17; in other words, a difference is made between »secondary partition in particles« in the formula X, and »tertiary division« in formula Y; further: quartary and quintary instead quaternary, etc.

The Greek spelling of names is also used, e. g. Arkatas, Plutarchos, Ptolemaios etc., and the spelling of the terms: Trichord, Tetrachord, Tritonos, Medium represent international spelling; and capital letters for »the tone Octave«, but small letters »the interval octave«.

2. To draw attention to 3 principles which have lately been set forth concerning systematic comparative Musicology:

a) that in the development of tone-systems from the earliest times to the present day there has, without a doubt, been a much greater stability than has hitherto been supposed, viz. Professor v. Hornbostel's demonstration of the stability in the length of the flute (and the gold measure) from 2000 up to 3000 B. C. till far down through the ages (Note 1). This stability we here transfer to the proportion between the pitch of the tones;

b) that in mentioning the ancient tone-systems it is only possible within very narrow limits to rely upon abstract Fifth-steps, but that one can rely in the highest degree on the concrete **shortening of a string** (secondary »partition into particles« formula X) cf. v. Hornbostel's remark: »One of the fundamentals of the Indian tone-system, the partition of a string, is evidently primitive« (Note 2); and v. Hornbostel's and R. Lachmann's observation in 1933: »Breloer sought to deduce Bharata's (Indian) system solely from this principle (the pure Fifths); he arrived at the 22 Sruti through a chain of pure Fifths. But it is not clear why the advance of the Fifths should stop short at 22 Sruti.« (Note 3).

c) that the principle of **Relativity** can be recognized also in relation to the tone-intervals, steps, cf. v. Hornbostel's declaration: »The investigations of recent years have led to surprising consequences. As is known, melodies can be displaced in their pitch as optical figures in space, without changing their shape; they may also be enlarged or diminished as figures in space without alteration of their shape if only the **relative** interval-distances are kept. In this way the structure-building intervals, as the Fifth and the Octave, also keep their function in the new formation.« (Note 4).

Indeed these 3 principles can also operate in a wider sense, as the above-mentioned authors also point out: »Thus the comparison of musical styles as well as the comparison of other musical functions — tones and instruments — might prove a valuable aid in the investigation of the history of civilization«. (Note 4).

For example: In his investigation of the length of the flute (and of the gold-measure) Professor v. Hornbostel has built up a hypothesis of an ancient **culture-stream** from China across the Pacific Ocean to Central America.

3. To add to these 3 principles the under-mentioned system of acoustic **methods of work**:

Chapter I: 5 kinds of exact interval-calculation, and:

Chapter II: 7 hypothetical principal-rules for the origin (genesis) and structure of tone-systems.

4. To make plain the advantage of having an objective means of **valuation** for ancient as well as modern or future tone-systems, which, mathematically speaking, can only occur by means of the **authoritative** golden system formed by the golden cut

(super division) of the octave according to the formula: $\frac{1}{\omega} = \frac{\omega}{1-\omega}$ or

$$\text{Formula: } \frac{\omega^2 + \omega}{\omega^2 - \omega} = 1 \quad \text{or} \quad \frac{\sqrt{5}-1}{2} = \omega \text{ (Omega),}$$

in decimals: $0,381,966 + 0,618,034 = 1$

Luca Pacioli's »Divina proportione«, Kepler's »sectio divina« and »gemma« i. e. precious stone (Note 13), the everlasting fundamental formula for universal power of **adaptation**, — here the formula for universal **sense** of harmony.

CHAPTER I

FIVE KINDS OF EXACT INTERVAL-CALCULATION.

Scheme 1. shows five methods of calculation of the intervals:

Group A. Quantitative string-partitions (Nos. 1-2), the musical faculty vision (musikalisches Sehen).

1. Primary (Flageolet)-string-partition, i. e., Partial- or Flageolet-tones are created by partition of a string (or flute) into several equal parts, which all vibrate at the same time, e. g. Natural triple chord c e g c', the Partial-tones Nos. 4, 5, 6 and 8.

The difference in the first grade between the cents of the adjacent Partial-tones approximates to the golden tones (Chapter IV).

The difference in the second grade approximates to the golden Molecules:

Partial-tone	Cents	1st grade	Golden tones:	2nd grade	Golden molecules:
No. 3 g	701.955	498.05	503.79 f	111.7	118.9 des
— 4 c'	0.	386.31	384.86 e	70.7	73.5 cis
— 5 e'	386.314	315.64	311.36 es	48.8	45.4 deses
— 6 g'	701.955	266.87	265.93 dis		
— 7 ais'	968.825				

2. Secondary (ornamental, decorative) string-partition: only part of the string vibrates; we divide the string into 720 particles = $3 \times 3 \times 4 \times 4 \times 5$.

The tone-fractions are indicated in the following »stable formula X«:

{the whole string as the common numerator}
 {the vibrating part as the denominator} never inverted.

Plutarchos' median point, the secondarily halved octave, is the Fourth, the Mese, Indian Mahijama: $\frac{720}{540} = \frac{4}{3}$ in formula X: $\frac{4}{4 \cdot 3 \cdot 2}$ with the common numerator

The octave secondarily 3-parted is the small Third es = $\frac{720}{600} = \frac{6}{5}$ and the Fifth g = $\frac{720}{480} = \frac{3}{2}$, Plutarchos' Paramese, Indian Pancama, (para = i. e. »beyond« the Mese) in formula X: $\frac{6}{6 \cdot 5 \cdot 4 \cdot 3}$ with the common numerator 6.

Tautological: the Fifth secondarily halved is the tone es: $\frac{6}{6 \cdot 5 \cdot 4}$.

Group B. Qualitative divisions, (Nos. 3-5).

3. Tertiary (arithmetic) division of the **decimals** of the tone-fractions (as decimal-fractions), or division of the difference between the vibration-numbers, also indicated in the following »stable formula Y«.

The octave 2,0 tertiary halved is just the Fifth $\frac{3}{2} = 1,5$ with the common denominator 1, or in formula Y: $\frac{2 \cdot 3 \cdot 4}{3}$ with the common denominator 2, see Scheme 19.

The Fifth = 1.50 tertiaryly halved is e, either 1.25 (0.50 halved) or in Y: $\frac{4 \cdot 5 \cdot 6}{4}$, with the common denominator 4.

The number of vibrations (by Dr. Illo Peters called **objective tone pitch**) increases proportionately to the given decimal-fractions.

The difference between the decimal-fractions (or vibration-numbers) of 2 adjacent Partial-tones is constant 0,25; $\frac{3}{2}$ minus $\frac{5}{4} = \frac{1}{4}$; or g' minus e' = C, compare Rule 5, d.

4. Quartary (geometrical, neutral) logarithm-division, or division (and transposing) of tone-intervals, steps; subjective tone pitch.

The product of 2 tone-fractions is consistent with the sum of their logarithms; for example: 2 Fifths-steps $\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$ agree with

$$\begin{cases} 2 \times 176.0913 = 352.1826 \text{ as logarithm,} \\ 2 \times 701.9550 = 1.403.9100 \text{ as cents, minus } 1200 = 203.91 = d + \end{cases}$$

Professor Joseph Yasser, New York, remarks »that the New York Public Library possesses a very rare and original manuscript: The Geometrical Scale in Musick, or Gam-Ut reduced to Geometrical proportions, and according to the statement of its author (whose name Gaudy is merely guessed from indirect indication), was written some time before the year 1705« (Note 5), — he was thus the pioneer before the Swiss mathematician Leonhardt Euler (1707—83), who in the year 1729 limited the common logarithms of tone-fractions to one octave (2) by dividing them by the logarithm of 2 = 0.30103.

In order to facilitate the transition to Ellis' cents as an international method of counting, an easy and practical method of logarithm-reckoning was introduced by me in 1929 by using the Constant K, i. e. (Scheme 22):

$$\begin{array}{rcl} \text{Log. } 1,2 = \dots & 0.079.1812 \\ \text{minus log. of } (\log. 2 = 0.30103) = \text{minus} & 0.478.6098 + 1 \\ & \text{Difference K} = 0.600.5714 \end{array}$$

Thus an addition and a subtraction are merged into one single addition = K.

5. Quintary structure is used in connection with the stretching of a string, see Scheme 1 above, $(\sqrt[5]{2})^2 = 2$ (or, the octave: $2^2 = 4$).

By pressing the string down on the finger-board (e. g. on a violoncello) the string will be somewhat tightened, and, accordingly, the pitch of the tone will also be somewhat raised (quintarily).

Resumé: a) The methods of work set forth in Nos. 2, 3 and 4 will be recognise in the Greek splitting of fractions, formula Z, e. g., the Fourth divided by three
 $\frac{4}{3} = \frac{12}{9} = \frac{12}{11} \times \frac{11}{10} \times \frac{10}{9}$ of which:

Form.			Cents
X	1st fraction	$\frac{12}{11}$ is secondary 3-partition common numerator 12.	150.6
....	2nd fraction	$\frac{11}{10}$ equals in cents: only little below quartary 3-division $= \frac{498}{3} =$	165.0 166.0
Y	3th fraction	$\frac{10}{9}$ is tertiary 3-division common denominator 9.	182.4

The quartary cents are but little below the average number of the secondary and tertiary cents, 166.5 cents.

Should the Fourth be divided by 4, the result will be:

Formula Z.	$\frac{4}{3} = \frac{16}{12} =$	$\frac{16}{15} \times \frac{15}{14} \times \frac{14}{13} \times \frac{13}{12}$	of which:
resp. Formula.....	X	Y	
Cents	111.7	124.5	
$\frac{1}{15}$ is secondary 111.7 cents, common numerator 16, X, $\frac{13}{12}$ - tertiary 138.5 - , - denominator 12, Y,			

Whereas the average number between $\frac{15}{14}$ and $\frac{14}{13}$ comes close to the quartary cents 124.5

b) **Twofold symmetrical calculations.** Pythagoras may have established his small Thirds $= \frac{32}{27} = 294.1$ cents by means of

secondary 8. partition	$\frac{32}{32 \quad 27 \quad 24}$	our X.
Spaces 8	$= 5 + 3$	
tertiary 9. division	$\frac{27 \quad 32 \quad 36}{27}$	our Y.
Spaces 9	$= 5 + 4$	

Or by means of particles and decimals respectively:

secondarily	c es — f	
Our particles.....	720 607 $\frac{1}{2}$ 540	
Spaces abbreviated	$112\frac{1}{2} + 67\frac{1}{2}$ $5 + 3$	= 180 = 8
tertiarily.....	c es — f	
Decimal-fractions	1,0 1,185 1,333	
Decimals abbreviated	0,185 0,148 5 4	= 0,333 = 9

c) The »Natural quadruple chord«:

Partial-tones Nos. 4—8	c e g [als] c'	
Primarily: Fractions	1 $\frac{5}{4}$ $\frac{3}{2}$ $\frac{7}{4}$ 2	
Secondarily: our particles, quickly decreasing spaces	720 576 480 411 $\frac{5}{7}$ 360 144 + 96 + 68 $\frac{4}{7}$ + 51 $\frac{5}{7}$ = 360	
Tertiarily: numerator in Y	4 5 6 7 8	Nos. 4—8
Quartarily: our cents, slowly decreasing intervals	0 386 702 969 1200 386 + 316 + 267 + 231 = 1200	
Quintarily; stretching, 2nd power of No.	16 25 36 49 64 ... 9 11 13 15 = 48	
Increasing tension		
Arithmetical progression, Andr. Kornerup's observation	2 + 2 + 2	= 6

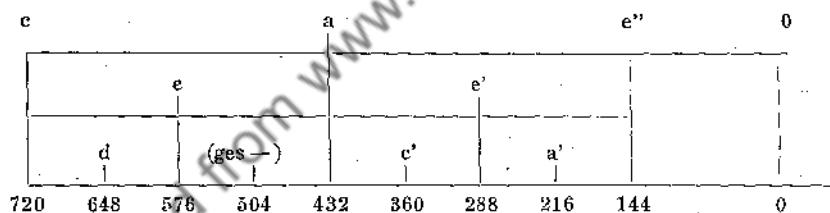
CHAPTER II

SEVEN HYPOTHETICAL PRINCIPAL RULES.

The following rules are proposed in the hope that a cooperation between philologic-historic inquiry and instrument-investigation on the one hand and theoretic-acoustic calculation on the other will lead to a clearer understanding of the origin and development of the ancient tone-systems than has hitherto been the case. Naturally, these rules are partly hypothetic, and are only advanced as a basis for discussion.

Group A. Presumable Construction of Scala-types and Tonics, (Rules Nos. 1—3).

Principal Rule No. 1. The stepwise progression of a scale-type is formed by secondary partition (on the string). 5-partition of the whole string gives thus (by three successive halvings) »the ornamental pentatonic scale«: $c - d = \frac{10}{9}$ $e - (ges -) = \frac{10}{7}$ $a - c' = \frac{10}{9}$



Secondary 20-partition of the whole string, then partly secondary 5-partition of the Fourth, gives also 5 ornamental Tetrachords in oriental tuning:

1	$\frac{20}{19}$	$\frac{10}{9}$	$\frac{20}{17}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{10}{7}$		
c	des —	...	es —	...	f	...	Doric	Primitive Greek before Pythagoras?
c	...	d	es —	...	f	...	Phrygian	
c	...	d	...	e	f	...	Lydian	
c	...	d	...	e	...	(ges —)	Tritonos	
c	des —	e	f	...		Harmonic, oriental.	

for instance: the Greek »scale-types«, Scheme 17, Nos. 1—7.
 Jewish Hedjaz-Kar, double harmonic.... — —, — 8,
 — Ahavad-Rabbah, harmonic-Doric. — —, — 12,
 according to some indications of A. Z. Idelsohn (Note 6).

Principal Rule No. 2. The oriental sequence of Tonics (key notes) can be formed by tertiary division. Three consecutive halvings of the Octave, Fifth and Third give thus the presumed primitive Greek (before Pythagoras) as well as the Indian fundamental Tonic-sequence: $c \left| \frac{9}{8} = d + \right| e g c'$, (only Partial-tones):

	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{8}{5}$	$\frac{27}{16}$	$\frac{15}{8}$	2	$\frac{9}{4}$
1st halving	c			g			c'	
2nd halving	c	e	g		b		d +	
3rd halving	c	d +	e	g	a +	b	d +	

The result, the **oriental (Indian) Tonic-sequence:**

Roman numerals:	I	II	III, IV	V	VI	VII, I.		
Oriental Tonics.	c	d +	c, f	g	a +	b, c'		
Indian Sruti-No.	0	4	7.9	13	17	20, 22		
Scale-Structure:	4	3	2	4	4	3	2	Ma-Grama
Scheme 2, J:		—	—	9	4	—	9.	= 22

In any case it agrees, mathematically, exactly with what Victor Mahillon (cf. E. v. Hornbostel and R. Lachmann's report, Note 7) has reported (without mentioning the source) as an old Indian prescription for the partition of an F string, so that seven tones can be made in the middle third part of the string corresponding to the number of particles on a whole string as given below (transposed from an F string to a C string by multiplying by $\frac{3}{2}$):

F. 480 particles	426 $\frac{2}{3}$..	384 .. 360 320 .. 284 $\frac{4}{9}$.. 256 .. 240
C. 720 transposed	640 ..	576 .. 540 480 .. 426 $\frac{2}{3}$.. 384 .. 360
C. Ma-Grama, Major:	d +	e f	g a + b c'
			F. Particles 288 270 240
			C. transposed 432 405 360
			C. Sa-Grama, Lydian-Phrygian: a bes — c
			Fractions: $\frac{5}{3}$ $\frac{16}{9}$ 2

The difference between Ma and Sa has reference to the structure of the two Indian scale-types respectively, see Rule 4.

Principal Rule No. 3. The Pythagorean and Persian Tonic-sequence is naturally formed quartarily, 6 Fifth-steps (Q Fifth, q Fourth):

	0	2Q	4Q	1q	...	1Q	3Q	5Q	-
Pythagorean Lydian scale.....	e	d +	e +	f.....	g	a +	b +	c'	
23 Pythagorean Instrument-tones: Structure	4	4	1		5	4	4	1	
19 — Song-tones —	3	4	1		3	3	4	1	
Compare: 17 Persian Sruti —	3	3	1		3	3	3	1	

Group B. Probable Construction of Ancient Tone-Material (Rules Nos. 4-7).

Principal Rule No. 4. Permanent and variable tones.

a) The transposing of secondary (ornamental, decorative) scales on the Tonics (key notes) mentioned may give several tone-systems with not a few common permanent tones, for instance the presumed 13 oriental permanent tones, and the 17 Persian tones with the exception of d, e, a and b:

Oriental Tonic No.	I	II	III	IV	ges —
Name.....	c	des —	d +	es —	e e + f ges 2 — ...
Oriental Tonic No.	V	VI	VII	I	
Name.....	g	as —	a +	bes —	b b + c' Scheme 2.

The tone ges 2 — e almost fis +.

The 13 Persian tones (with the exception of d, e, a and b) are presumably the common foundation for primitive Greek, Persian, Indian and the later Arabian tone-material. Continued transposition on other Tonics or with other scale-types gives new tones, varying both for the different tone-systems and within the same system, called »variable tones of the system«. The following are cited as examples:

13 + 4 = 17 Persian Sruti Nos., of which 13 are permanent, Scheme 2, P.

13 + 9 = 22 Indian Sruti Nos., of which 17 are permanent, Scheme 2, J.

22 + 2 = 24 medieval Arabian tones.

The 4 variable tones in Persian music are presumably the Persian Sruti Nos. 2, 5, 12 and 15; the 5 variable tones in Indian music: the Indian Nos. 1, 6, 10, 14 and 19.

The structure of the Tetrachords:

	C. Lydian				C. Phrygian				
Indian	Ma: g	a +	b	c'	Sa:	g	a	bes—	c'
Intervals	204	182	112		182	112	204	= 498
Sruti	4	3	2		3	2	4	= 9

b) The idea in itself of a few permanent Sruti-distances as a structure cannot be thought of as rising through a simple advance of a Second only, i. e.

	c	d+	e	f	g	
Indian Nos	0	4	7	9	13	
Ma, Lydian	4	+	3	+	2	= 9
Sa, Phrygian	→	3	+	2	+	= 9
Persian Nos	3	4	6	7	10	13
Doric			e+	f	g	a+
Transposition	d+	es-		f	g	= 7

but tertiary 9-division of the Fourth gives also the Indian Sruti-numbers for the upper Phrygian Tetrachord in Sa, Lydian-Phrygian.

Common denominator 48:	d+	e	f	g	
	$\frac{1}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{7}{2}$	
Numerators	54	60	64	72	
Distances	6	+	4	= 18
abbreviated, Indian Sa	3	+	2	= 9

corresponding to the tertiary 9-division.

c) The number of touches: Scheme 2 P, in all $7 \times 7 = 49$, and Scheme 2 J, in all $5 \times 7 \times 2 = 70$ disperse naturally very unevenly (the columns in the middle).

The number of Molecules (intervals of equal value) is, according to all Sruti:

in Scheme 2, Persian		Total	In Scheme 2, Indian 7 \times 4 Commas	
12 \times 4 Commas		53	5 \times 3	—
5 \times 1		Commas	10 \times 1	—
.....	2 Molecules		3 Molecules
Intervals 17		Intervals 22

This condition can naturally be changed by continued transposing. The expression "Molecule" for equal intervals between tones in stepwise progression was suggested in 1923 by Professor Chr. Kromann, (1846—1925), Copenhagen.

Principal Rule No. 5. Chromatic and Enharmonic.

a) In the practice of Music even Pythagoras has not been able to carry through the "consequent" Pythagorean Fifth-system, but he has had to content himself with the 7 fundamental diatonic Tonics, and has presumably formed the intervening chromatic and enharmonic tones by secondary interpolation, for instance, the above mentioned chromatic des —, es —, etc. by secondary halving of the small Second c. . d, or d+. . e, whereby with great approximation the same interval is obtained from des — to d + = 113.7 cents, and from e to f = 111.7 cents — inaudible difference (Formula X):

1) c . . d	$\left\{ \begin{array}{c} 20 \\ 20 \quad 19 \quad 18 \end{array} \right\}$	$\text{des} = \frac{20}{19} = 88.8 \text{ cents}$	Inaudible difference 1.4 cents, nearly k, Scheme 14.
2) Pythagorean: 5 Fifths below		$\approx \text{des} = \frac{256}{243} = 90.2$	
3) c . . d + $\left\{ \begin{array}{c} 18 \\ 18 \quad 17 \quad 16 \end{array} \right\}$		$= (\text{des} \frac{1}{3}) = \frac{18}{17} = 98.9$; c . . es 3-parted	
4) syntonic, difference, Scheme 24, between the Partial-tones Nos. 16 and 15		$= \text{des} = \frac{16}{15} = 111.7 \text{ ct.}$	Inaudible Seisma-difference: 2.0 cents.
5) Pythagorean: 7 Fifths up		$= \text{cis } 2 + = 113.7$	
6) Golden des = $5 \times 503.8 \text{ cents}$		$= \text{des} = 118.9 = \text{c . . d super divided.}$	
7) c . . e	$\left\{ \begin{array}{c} 15 \\ 15 \quad 14 \quad 13 \quad 12 \end{array} \right\}$	$= (\text{des}) = \frac{15}{14} = 119.5$, c . . e 3-parted.	

The Greeks Eratosthenes and Plutarchos used the secondary fraction 20/19; Arkytas, Didymos and Ptolemaios 16/15; Plutarchos also 15/14 in *Kroma malakon* (soft, i. e. diminished *Kroma*).

When construing tone-systems one can for convenience overlook the small inaudible differences and, on paper, calculate by theoretical fractions. The Greeks themselves have to a large extent made use of the splitting of fractions, Formula Z, i. e., $d = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$, of which { the first is secondary halving and the second a tertiary one.

By this primitive method of work, splitting of fractions, the legitimate tone systems have been far outstripped, by continual partition, i. e., secondary interpolation; thus Eratosthenes and Plutarchos halve des = $\frac{20}{19} = \frac{40}{38} = \frac{40}{39} \times \frac{39}{38}$, of which the first is a secondary enharmonic deses = $\frac{40}{39} = 44$ cents, while Didymos halves the syntonic des = $\frac{16}{15} = \frac{32}{30} = \frac{32}{31} \times \frac{31}{30}$, of which $\frac{32}{31} = 55$ cents also is the secondary enharmonic deses, just 11 cents larger.

Curiously enough Arkytas makes use of the fraction $\frac{28}{27} = 62.9$ cents, very nearly the Molecule in the 19-toned temperament 63.16 cents, just the Fourth tertiary 9-divided (Formula Y):

$$\begin{array}{cccc} c & \text{des} = & \text{es} = & f \\ 27 & 28 & 32 & 36 \\ \hline & & 27 & \end{array}$$

with $\frac{28}{27} = \text{des} = 62.9$ cents and $\frac{32}{27} = 294.1$ cents, see Chapter I, Resumé, b.

Plutarchos has a similar fraction $\frac{30}{29} = 58.7$ only a little lower, and $(\text{des}) = \frac{15}{14} = \frac{30}{28} = 119.5$ cents, syntonic d secondarily 3-parted, Formula X:

$$\begin{array}{ccccc} & & 30 & & \\ & 30 & \hline & 29 & \\ & & & 28 & \\ & & & \hline & 27 \end{array}$$

Ptolemaios uses the fraction $\frac{45}{44} = 38.9$ cents, very nearly the Molecule in the 31-toned temperament, 38.71 cents, the Pythagorean d + secondarily 5-parted (Formula X):

45	44	40
----	----	----

b) Interpolation within the **Tetrachords** is also presumably to be found to a great extent among other nations than those mentioned. Thus Helmut Ritter quotes from Rauf Bey (Note 8) for the **Turkish scale-types** »Suznak« and »Neu eser« one and two Tetrachords respectively of two unusual forms which we shall call:

Scheme 17, No. 13.

{ «uneven Tritonos-Tetrachord» 204 + 120 + 267 ≈ 591, Medium 111 «uneven harmonic» 120 + 267 + 111	cents
	total 702 total 498

Scheme 17, Nos. 10 b and 13.

which however, can easily be explained in the following way:

The interval on the string »d + fis +« (in Tritonos) is parted secondarily into 3 parts or the Fifth »d+ ... a+« into 5 parts in:

Particles:	d +	es + $\frac{1}{3}$ Comma	fis +	a +	cents
	640	597 $\frac{1}{3}$	512	426 $\frac{2}{3}$	
Spaces	$42 \frac{2}{3}$	+	$85 \frac{1}{3}$	+	$85 \frac{1}{3}$
Abbreviated	1	+	2	+	$\approx 213 \frac{1}{3}$ ≈ 5

Thus the tone $\frac{720}{597 \frac{1}{3}} = \frac{135}{112} = 323.4$ cents, $\frac{1}{3}$ Comma over es, which is 315.6 cents. The two scales can hereafter be characterised as follows:

Suznak	c d + e f	cents	g (as) b c	Scheme 17 No. 10 b.
	Lydian Major.	204		
Neu eser	c d + (es) fis + uneven Minor Tritonos	111	g (as) b c uneven harm. repeated	No. 13

c) In the same way some five- or six-toned scales, as for example some Indian scales may be explained by Trichords:

Trichords:	Secondary partition:		Middle tone:	
	Particles	Abbreviated		
c d f	72 + 108	2 + 3 = 5		10/9
c d + f	80 + 100		4 + 5 = 9	9/8
c (es—) f	108 + 72	3 + 2 = 5		20/17
c es — f	112 $\frac{1}{2}$ + 67 $\frac{1}{2}$		5 + 3 = 8	32/27 *)
c es f	120 + 60		2 + 1 = 3	6/5
c c f	144 + 36	4 + 1 = 5		5/4

*) see Chapter I, Resumé, b.

And it may be supposed that many other riddles of a similar type can be easily cleared up as being something quite natural by secondary interpolation alone — without fantastic Fifth-steps.

d) We might probably solve Professor v. Hornbostel's theory concerning **Comma-Fifth** and **Comma-Fourth**, probably g— and f+ respectively, by means of

1) **secondary interpolations**, »4-partition of 2 Fifths«, d..a and d+..a+ respectively:

		Formula X:			
		40			
		36	30	27	24
g—		d	f	g—	a
		648	540	486	432
		2	1	1	
3 proportionally abbreviated spaces		2	1	1	
		640	533 $\frac{1}{3}$	480	426 $\frac{2}{3}$
f+		d+	f+	g	a+
		27			
Formula X:		24	20	18	16
Then Comma-Fifth:		d..... a 4-parted	or: f	a	} halved
— — -Fourth:		d+ g 3 —	or: d+	a+	

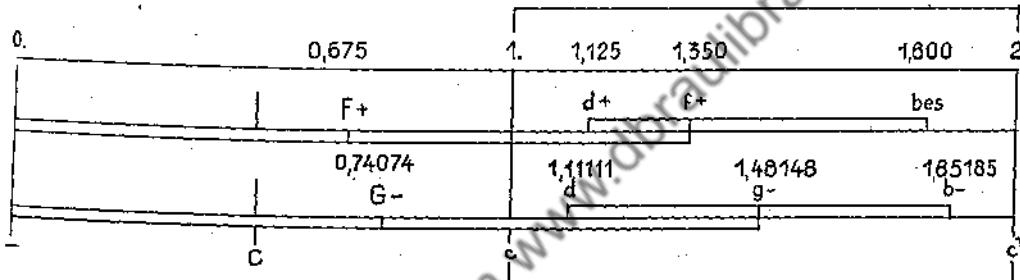
2) Further Tertiarily following divisions, see **Scheme 19**:

3-divided	4		8		
	10		5			
Formula Y	30	32	36	40	45	48
Denom. 27: d es — f g — a bes — b —						
Halved	10		10		
		8		8		
3-divided	18		36			
5- —	18		27			
Formula Y	90	96	108	120	135	144
Denom. 80: d + es f + g a + bes						
Halved	12		12			

3) Further »bes ... d+ ... f+« and »g— ... b— ... d« are Major triple-chords:

$$\left\{ \begin{array}{l} f+ \text{ is deep difference-tone} \\ g- \end{array} \right| \begin{array}{l} \text{between } d+ \text{ and bes} \\ - \quad d \quad \text{and } b- \end{array} \right\}$$

Super-Tonic bes = $\frac{9}{5} = \frac{72}{40} = 1,800$	resp. b— = $\frac{50}{27} = 1,85185$ Third
minus Third d+ = $\frac{9}{8} = \frac{45}{40} = 1,125$	minus d = $\frac{30}{27} = 1,11111$ Sub-Fifth
Deep difference F+ = $\frac{27}{40} = 0,675$ G— = $\frac{20}{27} = 0,74074$
The Fifth f+ = $\frac{27}{20} = 1,350$ g— = $\frac{40}{27} = 1,48148$ Tonic



Finally it may be proved by means of other similar difference-tones. Prof. v. Hornbostel however, gives as a physical explanation of this problem (Note 8), (f+) = 521.5 cents, (g-) = 678.5 cents, die Blas-quarte und Blas-quinte in decimalfractions: 1,3515 and 1,4798 respectively.

Principal Rule No. 6. The intervals between the tones in stepwise progression can often be grouped in several »Greek Triads« (3 neighbour tones) of two dimensions in two directions:

great »Triad«	direct 4 + 1 Comma	retrograde 1 + 4 Commas, Pythagorean
small »Triad«	» 3 + 1 »	» 1 + 3 » Arabian?

As it happens the octave comprises 53 Commas of 22.24 cents (about midway between the syntonic and Pythagorean Comma, respectively 21.50 and 23.46 cents); from Scheme 3 can, for example, be constructed Scheme 4 as an experiment: the »direct Greek Triads« with 53-toned tone-No. (Nicholas Mercator, Dane, 1675):

Perhaps the structure of the »Greek Triads« has had some importance in Melody formation.

Principal Rule No. 7. Javanese salendro and pelog Gamelan were interpreted by F. Lassen Landorph (Note 9) in 1923 as »Two tone-systems, in which two Gamelans (adapted to two different systems) cannot be played together«; we assume that they have both been built up by quartary interval-division, but in two different ways. During his stay in Sumatra (1877—99) Lassen Landorph recorded the vibration-numbers (oscillations) for the tones, and arrived at the conclusion that

1) the older form, the Salendro-octave, has 5 tones (Pentatonic) »with a distinct tendency towards five equal (quartary) intervals« within the octave, thus a 5-toned temperament; whilst

2) the latter form, the Pelog-octave, is also »called 5-toned, as the whole material of 7 tones in several places is seldom or never used, but on the other hand, various combinations on 5 tones only«.

Following the vibration numbers recorded by Lassen Landorph, **Scheme 5** has been constructed showing the Salendro-octave as a 5-toned temperament and the 7 tones of the Pelog as tone-material, i. e.

partly »a harmonic Tritonos«, in quartary 4-division with omission of the central tone: »c cisis ... eis fis +«,

partly a Pythagorean harmonic Tetrachord, »g as— ... b + c'«, in quartary 5-division with omission of the central tone. The interval »as— ... b+« is the syntonic es = 6/5.

Other intervals:

Nos.	Interval	Nos.	Inter.	Nos.	Inter.	Nos.	Inter.	Nos.	Inter.
1—2	cisis	4—5	des	5—6	des—	6—7	es	4—6	d +
3—4	cisis	5—6	des—	6—7	es	7—8	des—	6—8	e +
2—3	Sum es —	Sum	d +	Sum	e +	Sum	e +	Sum	ges —

Lassen Landorph is of opinion that the Salendro is reminiscent of the very ancient Chinese Tone-system, and »in its most simple composition, with reference to the Java-tradition, it is supposed to derive from the oldest Hindu period, or, more correctly, to be traced to this period, which means, from the beginning of the Christian era«; — while »Pelog must be presumed to be later, as it, with its various tones and varied musical instruments, is more fully developed«. Both are, however (partly) built up on quartary interval-division, — naturally secondary in practice.

The scale: »g as— b+ c fis+ g« may be presumed to be pure Pythagorean (fis+ = ges 2—).

In the pelog Gamelan the tone Pelog, 512 particles = 590 cents, may however easily be found on a siring (or on the flute) with great approximation by the 5-partition of the octave, and the Fifth g exactly by halving, while analogously the other tones may be found by secondary 10- and 5-partition respectively, as shown in Scheme 5.

CHAPTER III

THE PYTHAGOREAN SYSTEM IN THEORY AND PRACTICE.

That the pure, i. e. consistent Pythagorean system can only be taken symbolically is evident partly by the fantastic tone-fractions formed in the outer circle of the system, partly by the Greek Schisma, for instance

Pythagorean tones:

$$9 \text{ Fifths} \text{ is } \text{dis } 3 + \text{e} = \frac{19.683}{16.384}$$

$$10 \quad - \quad \text{ais } 3 + \text{e} = \frac{59.049}{32.768}$$

$$11 \quad - \quad \text{eis } 3 + \text{e} = \frac{177.147}{65.536}$$

The fractions: overstated

Syntonic tones:

$$\text{but es} = \frac{6}{5} = 315.6 \text{ cents}$$

$$\text{, bes} = \frac{9}{5} = 1017.6 \quad \text{,}$$

$$\text{, f}+ = \frac{27}{20} = 519.6 \quad \text{,}$$

.... natural.

Only 6 Fifth-steps give the same tone as »2 Fifth-steps and in the opposite direction 1 syntonic Third«, that is:

$6 \text{ Pythagorean Q minus 3 octaves}$ $2400 \text{ minus } (2 \text{ Q} + 1 \text{ syntonic Third})$	$= 4211.730 - 3600$ $= 2400 - 1790.224$	$= 611.730$ $= 609.776$ $\text{Difference, a Greek Schisma} = 1.954$	$\hat{=} \text{ fis } 2 +$ $= \text{ges} -$ cents.
---	--	--	---

After Pythagoras having altered the presumed primitive Greek

Tonic-sequence (key-notes): c d+ e f g a b+ c' to his Diatonic (Persian) : c d+ e+ f g a+ b+ c'		
--	--	--

which is also the authorised Greek Tonic-sequence, he hereby reaches his limit, and beyond that he only creates syntonic tones with the exception of one Schisma, so that the Pythagorean exaggeration recoils, in that, beyond 6 Fifths, all other Fifths return like a boomerang, as syntonic interchangeable tones. It is the playful way of Nature that all exaggeration corrects itself.

The Arabian-Persian musicians probably knew, before 636 A. D. that, for instance, 8 q Pythagorean = fes 2 — are almost identical with the syntonic e, (according to Helmholtz, as quoted by Jonquière, Note 10).

If to this be added that the 4 chromatic tones des—, es—, as— and bes— are so near to the primitive secondary tones that the difference of 1.4 cents is inaudible

all Pythagoras's exaggerated tone-fractions may be changed to reasonable human fractions.

The Pythagorean system can thus (with the exception of the inaudible Schisma) be indicated as quoted in **Scheme 6**:

Song Lexis	Instrument Krusis	Scheme 14:
—	3	enharmonic interchangeable tones
2	2	chromatic
5	5	—
7	7	diatonic Fifth steps
5	5	enharmonic interchangeable tones
—	1	—
19	23	Tones in all in 3 stages

The tones, however, are not used with equal frequency; if all the Greek scale-types be played on the following 7 Tonics (key-notes): c d + e + f g a + and **bes** — for song, the number of touches will be $7 \times 7 \times 7 = 343$, distributed as the numbers in Scheme 6, with the maximum 37 on g, which is the middle one of the 7 Tonics selected, arranged according to Fifths.

The number of Molecules for 19 song-tones will be $10 \times 90.2 + 2 \times 66.7 + 7 \times 23.5$ cents = 1200 cents.

The names of the song-tones are thus given in the Greek **Triads** from above downwards; for the instrument-tones a name-No. was originally given to the uppermost in the Triad (Note 11) following our hypothesis as indicated in **Scheme 7** according to triple-chords (chords of Thirds, English triads).

Scheme 17 Nos. 1—7, shows the 7 Pythagorean scale-types in syntonic or golden tuning, 6 paired off symmetrically {Nos. 1 | 2 | 3
and 7 | 6 | and 5.

CHAPTER IV

THE AUTHORITATIVE TONE-SYSTEM, THE GOLDEN SYSTEM AS AN OBJECTIVE MEANS OF VALUATION.

However, in order to pass judgement on the various historical tone-systems, the chief defect of which is the displacement of **Commas** between for example »d and d+«, »des — and des« etc., it is necessary first to construct an authoritative tone-system **without** these Comma-displacements, which claims that many tones with plus would be made much lower, and many tones with minus much higher.

In August 1930 I found the new Fifth in two ways which gave the same result.

Group A. Arithmetic Series I—VI, retrograde and direct.

The sum of 2 golden adjacent Nos. forms the following No., and the sum of the cents of two golden adjacent tones forms the following cents in the same golden Series.

1. The astronomer J. Kepler (1571—1630) combined tones with the sum of the numerators and the sum of the denominators respectively of 5 tone-fractions:

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{8}{5}$	$\frac{13}{8}$	Partial-tone 840 5 cents
C	c	g	a	as	gisis	

The golden gisis..... 843.2 →

In the 19-toned and the 31-toned golden section respectively we transform the tone-sequence in this manner (see Scheme 10):

Fourth-	Series 1	retrograde:	19 toned	31 —	50 —	Double super division							
						1 cis	2 des	3 d	5 es	8 f	13 as
						1 deses	2 cis	3 des	5 d	8 es	13 f	21 as	
						1 bisis	2 deses	3 cis	5 des	8 d	13 es	21 f	34 as



Ludwig Sonnenberg, principal teacher in Bonn (1820–88) called the sequence 1, 2, 3, 5, 8, 13: »Kepler's Series« (Notes 12–13), our golden **Fourth-Series I retrograde**, in 1844 extended up to No. 17 by Gabriel Lamé, in 1929 up to No. 40 by L. Kaiser.

2. Dr. Ludwig Kaiser, the mathematician, found by pure mathematics (Note 14) (without reference to tones) a similar sequence; we call the latter »Kaiser's Series«, our **Fifth-Series II**, which we use in the following manner:

Fifth- Series II retrograde	Double super division			
	19-toned	4 dis	7 fes	11 g
31- —	7 dis	11 fes	18 g	29 ces

3. In May 1930 Andreas Kornerup, engineer, directed my attention to the fact that the Fifth in these systems has tone-Nos. which form similar sequences of Nos. for example $\frac{\text{Fifth}}{\text{Octave}} = \frac{7}{12}, \frac{11}{19}, \frac{18}{31}, \frac{29}{50}, \frac{47}{81}$ etc.

We name the sequence 12, 19, 31, 50, 81 the **Octave-Series III**, Andreas Kornerup's Series, the denominators of the fractions.

a) It occurred to me to calculate these fractions in cents, which I did, August 15th 1930. The value moved like a **pendulum** quickly approaching the point of balance, where the seventh decimal would be stable at the Fifth $\frac{1200 \times 3371}{6155} = 696.2145$ cents

(Acoustic statics).

The temperaments, the octaves of which are the denominators of these fractions, I called the **organic** temperaments, and, at the same time, it occurred to me to construct an **authoritative** »tone-system of the Fifth« with this Fifth which, however, also generally appears by means of the super-division (golden cut) of the octave, **Scheme 10**, Series III. I calculated the Fifth on August 17th 1930, as shown below:

b) The fraction of super-division	Cents	Series III:
$\varrho 0,61803398 \times 1200$ gives 741.64078	direct ases
$1 - \varrho 0,38196602 \times 1200 \dots 458.35922$	retrograde eis
Total 1,0	1200 cents	c'
Further { ases is 11 Fourths from c	and 4 octaves back	then f = 503.7855
eis is 11 Fifths — - -	— 6 — - -	— g = 696.2145
		Total = 1200 cents

In 19-toned and 31-toned golden sections respectively we use »Andreas Kornerup's Series« in the following manner:

Octave- Series III retrograde	Double super division			
	19-toned	0 c	7 eis	12 ases
31-toned	0 c	12 eis	19 ases	31 c'

The difference between golden eis = 458.4
and syntonic — = 457.0

Schemes 14 and 15: nearly $k = \frac{1}{15}$ Comma = 1.4

4. Out of the calculated golden system I have formed some other Series for instance 31-toned see **Scheme 20:**

				Double super division.		
Great Third-Series IV:	4 cisis	6 eses	10 e	16 ges	26 bes	
Tritonos — V:		9 feses	15 fis	24 beses	
Great Sixth— VI:		9 disis	14 geses	23 a	

Group B. Geometrical Constructions of golden Tones.

The golden tones and intervals can also be constructed geometrically as shown in **Scheme 8**, which indicates:

1) above, part of a regular **Pentagon**, the angle of which = 108° is divided into 3 parts by means of 2 chords; if the chord be equal to a Tetrachord — the interval C-F, (the Fourth f), this interval will be super-divided in »great double super-division«

$$\text{in } \left\{ \begin{array}{l} \text{C} \text{ --- D} \dots \text{Es} \text{ --- F} = \text{Series I} \\ 192.43 + 118.93 + 192.43 = 503.79 \text{ cents} \end{array} \right\}$$

2) below: a **square** placed inside a semicircle, whereby the diameter is super-divided into a »small double super division«

$$\text{in cents } \left\{ \begin{array}{l} \text{Cis} \dots \text{D} \text{ --- E} \dots \text{F} \\ \text{D} \dots \text{Es} \text{ --- F} \dots \text{Ges} \\ \text{B} \dots \text{C} \text{ --- D} \dots \text{Es} \\ 118.93 + 192.43 + 118.93 = 430.29 \end{array} \right\}$$

$$\text{corresponding with: } \left\{ \begin{array}{l} \text{C} \dots \text{D} \text{ --- F} \dots \text{G} \\ 192.4 + 311.4 + 192.4 = 696.2 \end{array} \right\}$$

3) further: within the square we can form a »great double super division«

$$\text{in } \left\{ \begin{array}{l} \text{D} \text{ --- Dis} \dots \text{Es} \text{ --- E} \\ 73.50 + 45.43 + 73.50 = 192.43 \text{ cents} \end{array} \right\}$$

Scheme 9 shows the golden system constructed by means of the **Pentagram** with the same chord C..F, so that the side is = C..Es in the Pentagon.

Fourth-Series I: C, Deses, Cis, Des, D, Es, F and further As, Des'.

Further: $\frac{\text{C. (Es)}}{\text{C..E}} = \frac{\text{es}}{\text{e}} = \text{Cosinus } 36^\circ$, the relation between small and great Third.

Scheme 10 shows »double super-division« in 4 Series; the upper and lower edges of a square are divided:

{ from e upwards into ases, direct,
— c' downwards into eis, retrograde.

If we continue a single super-division,

1) direct, upwards, we will get **decreasing** intervals upwards,

2) retrograde, downwards, we will get decreasing intervals downwards. We use the oblique lines { from the corner upwards on the right } forming 8 double super-divisions, which give the result:

	upwards, retrograde: Cents			downwards, direct, Scheme 10:
Series I	c — es ..	f — as	815	e — g .. a — c'
	c — e ..	ges — bes	1008	cis — f .. gis — c'
	c — fes ..	g — ccs	1126	d — fis .. as — c'
	c — cis ..	c'	1200	c — ases — c'

We continue double super-division only in Series I:

Series I:	{	c — des .. d — es	311	a — bes .. b — c'
		c — cis .. des — d	192	bes — b .. ces — c'
		c — deses .. cis — des	119	b — ces .. bis — c'

The tones are in pairs supplement-tones (making an octave together) for instance:

Series I	{ retrograde	deses	cis	des	d	es	f	as	c'
	direct	bis	ces	b	bes	a	g	e	c

Scheme 20 shows golden tones in 6 Series formed geometrically by means of parallel lines in a Pentagon, for instance:

The tones : Eis	F	Fis	Ges	G	As	A	C'	Des'
Series I-III: III.	I.....	I.....	II.	I.....	III.	I.		
— IV-VI:.....	V.	IV.....	VI.....					

Scheme 21 shows cents (to 4 decimals) for an organic 19-toned section of the infinite golden tone-system with two Molécules:

{ t = 73.501 cis, which is the smallest interval in this section,
 v = 45.426 deses, the greatest interval in the next organic section on 31 golden tones.

	v.	t.	cents
The Molecules in 12-toned golden section			
19- — — —	5 × cis + 7 × des	= 1200	
31- — — —	7 × deses + 12 × cis	= 1200	
	12 × bisis + 19 × deses	= 1200	

v. outside the section.

Group C. All Comma-displacements disappear.

Example: $d = \frac{10}{9}$ and $d+ = \frac{9}{8}$

merges into the golden tone $d = 192.43$ cents, approximating to the secondary halving of the string between d and $d+$, which in the Formula X gives:

	180			
.162	161	160		
			$d \dots \dots \dots d+$	
Particles 648	644.3	640		
cents	182.4	193.1	203.9	

or $\frac{180}{161} = 193.12$ cents, the difference is inaudible,

so that the golden tone can easily be found approximately on the string.

The tertiary halving gives a somewhat higher tone (Formula Y):

$$\frac{160}{144} \quad \frac{161}{144} \quad \frac{162}{144} \quad \text{or } \frac{161}{144} = 193.20 \text{ cents.}$$

The structure of the triple chord is often the golden cut directly put to use (x = great Third, y = small Third, (2) and (3) formations):

Triple chord:	Symbol	Intervals	Small Sixth with golden cut:
Major 2nd form	xy (2)	e g c	retrograde: $311 + 504 = \{$
Minor 3rd —	yx (3)	g c es	direct: $504 + 311 = \}$ 815 cents

For teachers of harmony the golden system can thus also be of use through its clear and logical construction; thus xyy (1) the Dominant quadruple chord e g bes will, on the 1st step, be resolved into xy (3) = the tonic triple chord on the 3rd step: (e) f a c' (on the Tonic f) by which means the tones e, g and bes glide either 119 or 192 cents, »c des« and »c d« respectively, systematically up or down, with the intervals in cents:

From	o	385	696	1007	1200.	Symbols
	c	e	g	bes	c'	xy (1)
Gliding in cents		\rightarrow 119 192	\leftarrow 192	\leftarrow 119	\rightarrow 192	
to	e	f	a	c'	xy (3)	
	o	504	888	1200		

Since all Comma displacements disappear in the golden system, one can therefore transpose any scale whatever on any golden tone as Tonic (key-note) with the **smallest possible number** of tones; the authoritative Golden tone-system represents in this field the formula for the principle of the **smallest activity**, Nature's **economic minimum** principle. It is the system with the smallest possible number of tones (acoustic Okology), an outcome of Nature's wonderful **power of adaptation**, — the supporting principle of all life in Nature.

Andreas Kornerup has called the fraction of the golden cut $0,618,034$ Omega, which is recommended as an international expression in the formula $\frac{1}{\omega} = \frac{\omega}{1-\omega}$ or: $\omega^2 + \omega - 1 = 0$ or: $eis + ases = c'$, Series I, or $458 + 742 = 1200$ cents, which we here designate, once and for all, as the authoritative principle of relativity in the field of acoustics: The essential formula for the **universal sense of harmony**.

Even if the composers do not know of this formula, the Danish Physicist H. C. Ørsted (1771—1851) is indeed right in saying: »The work of the composer is based on mathematics although in a **deeper** measure than has ever dawned upon us.« (Note 15). It will be the task of the future student of the theory of music to get to the bottom of this deeper-lying law of Nature, the golden cut, the super division — the basis of the future renaissance of harmonics.

CHAPTER V

TEMPERAMENTS.

Tone-systems with 1 Molecule only.

The Octave-Series III, Andreas Kornerup's Series, marks the boundary of the organic temperaments, i. e. the only rational, the only temperaments fit for use, namely the sequence: 12, 19, 31, 50, 81 — with 19-toned as the practical and with 31-toned temperament as the Standard-temperament, see **Schemes 12** and **18**.

How many tones will be required for pianos, organs etc. is indeed a question; but, for practical measures only the organic 19 and 31 are efficient; all the others are unworkable. This is the authoritative judgement passed on the matter in question.

Ceterum censco: If we wish to abandon the 12-toned pianoforte we can in no circumstances choose any other than the practical 19-tonic temperament, and for finer requirements the Standard 31- (or the 50-) toned temperament.

All the inorganic temperaments ought to be excluded.

Scheme 11 shows, by way of example, how the Fourth-Series I, Kepler's Series, is carried through logically, in 19- or 31-toned temperament, where the sum of cents for two neighbour tones gives the next tone in the Series, but is split in, for instance, the 24-tone system, which is therefore authoritatively considered to be unworkable.

Further: Dr. P. S. Wedell and N. P. J. Bertelsen, actuary, Copenhagen, in January 1915 proved, by means of «the method of the smallest squares», that the 19-toned is better than the 12-toned temperament, and that the 31-toned, again, is better than the 19-toned. To have this judgement expressed in numbers it can easily be calculated how much the single tone in the different temperaments deviate from the golden tones, and by summing up such deviations for 35 tones (the tones of the 7 white keys and ♭, ♪, 2♯ and 2♭) the result is found to be as follows see **Scheme 12**:

Deviation for	12-toned Temperament	1174 cents estimated at 100 %
19-	—	458 — thus 39 —
31-	—	174 — — 15 —
50-	—	67 — — 6 —

In comparison it may be quoted that in the pure consistent Pythagorean system, the collective deviation for the same 35 tones is 1780 cents or 52% greater than for the 12-toned temperament.

Among the numerous observations as to the change of the piano-temperaments we shall quote the following few selections:

Director Gotfred Skjerne says (1909): »We are indebted to the tempered tuning for enormous musical progress but the ear is, as a matter of fact, coarsened.« (Note 16).

Professor Dr. José Würschmidt showed 1920—28 »that a division of the octave into 18, 24 or 36 parts can represent no natural extension of our tone system, but that we have before us such an extension in a division of 19 steps. (Note 17).

Professor Joseph Yasser, New York, recommends the 19-toned temperament with the purpose: »to enrich our musical language, particularly when one takes into account the growing significance of the independent twelve-tone foundation in modern music. (Note 18).

Professor Louis Kelterborn, Neuchâtel, (Note 19) proposes, further, on practical grounds, that simultaneously with the introduction of the 19-toned temperament, the pitch of »the tone a* should be made a little lower, so that e may keep the same pitch as it has now.

Scheme 13 shows, geometrically, an equilateral hyperbole through the tones No. 5 in various organic temperaments forming the tones in the Golden Fourth-Series I, retrograde, (with approximate pitches of tone).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

CHAPTER VI

CHANGE OF THE SYNTONIC SYSTEM INTO GOLDEN TONES.

a) In Schemes 14 and 15 we have the syntonic system, built on Pythagorean Fifths (the Partial-tone = 3/2) in horizontal lines and the Thirds = 5/4 in diagonal lines with an angle of 60°, according to the proposal of the Japanese Shohé Tanaka in 1890.

The axis through »ses, c or gisis« in the schemes is called Zero-Axis, because the difference between the syntonic and golden seses (equalling 15 golden Fourths) is almost nil, just as is the difference between the corresponding supplement-tones, syntonic and golden gisis (equal to 15 golden Fifths), which is seen in the following table:

{		4 syntonic Thirds.....	give 1545.255 cents	
minus 1	—	Fifth, the Partial-tone 3/2 = g	701.955	—
}		gives syntonic gisis	843.300	—
15 golden Fifths minus 8 Octaves, golden gisis, is		843.217	—	
inaudible difference		0.083	—	
Further: syntonic Third, the tone 5/4 = c		386.314	—	
4 golden Fifths minus 2 Octaves: golden e.....		384.858	—	
Slight deviation		1.456	—	
1/15 of a syntonic Comma 21.5062 cents is k =		1.434	—	
inaudible deviation		0.022	—	

All the tones in Scheme 14 can thus, in a practical way, be changed to golden tones by adding or subtracting a number of »k«. These numbers are regularly grouped in all the lines which can be enclosed through the tones in the Scheme, i. e. with regular rise in numbers of k, so that the Scheme will be reminiscent of mathematical number-designs, or **number-figures**, constructed by the Danish astronomer Thorvald Nicolai Thiele (1838–1910) in 1872 (Note 20).

Scheme 14 is thus divided into 9 symmetrical figures, each comprising 15 tones, formed on about 9 central points of, respectively + 15 or 0 or - 15 k:

	+	Zero	-
gisis —	gisis 0	gisis +	
+ 15	0	- 15	
c —	c	c +	
+ 15	0	- 15	
feses —	feses	feses +	
+ 15	0	- 15	
resp.	from + 22 to + 8	from + 7 to - 7	from - 8 to - 22

Thus, outside the Zero-Axis the tones are here changed by »addition on the left side«, »subtraction on the right side« of fifteenth parts of a Comma, increasing evenly in all directions.

Axis:	Direction:				Result: Central-points
1) Pythagorean Fifth-Axis, horizontal ... then one Small Third-step	to the right downward	o. c	- 4. g	- 8. d +	- 12. a + - 3. — 15. e +
2) Pythagorean Fifth-Axis, horizontal ... then one Great Sixth-step	to the left upward	o. c	- 4. f	+ 8. bes	+ 12. cs — + 3. + 15. c —
3) Great Third, Axis, 60° upward ... then one Great Sixth-step	to the right — left	o. c	- 1. e	- 2. gis	- 3 bis + 3 + 0 gisis
4) Small Sixth-Axis 60° downward ... then one Small-Third-step	to the left — right	o. c	+ 1. as	+ 2. fes	+ 3 deses - 3 0 feses
5) Small Third-Axis through the figure .	from + 15 gisis — (through > o. c) to - 15 feses +.				

If all the Third-lines, inclining 60°, be elongated they will cut the Zero-Axis in nothing but Zero-points, — ad. infinitum.

Two examples of the division of 1 Comma will finally be quoted:

$$\begin{array}{l} d + \text{sunk } 8 k = 11.4700 \\ d \quad \text{raised } 7 k = 10.0362 \\ \hline \text{Total } 15 k = 21.5062 \end{array}$$

$$\begin{array}{l} es \quad \text{sunk } 3 k = 4.3 \text{ cents} \\ es \quad \text{raised } 12 k = 17.2 \quad — \\ \hline 5 \times 3 k = \text{Total } 15 k = 21.5 \quad — = 1 \text{ Comma.} \end{array}$$

The golden $d = 192.43$ cents is only a trifle below the secondary halving of the distance on the string between d and $d +$, as above mentioned (Chapter IV, Group C).

Analogously the distance on the string between Pythagorean es — = $\frac{32}{27}$ and the syntonic es = $\frac{6}{5}$ is secondarily 5-parted according to the Formula X:

480		
	405	401
	es —	golden
with the particles	607,5	601,5
		600

An extreme example:

Pythagorean 6g = ges 2 — = 588.3 cents	Syntonic fis + = 590.2 cents
Scheme 14: plus 24 k = + 34.4 —	Minus 9 k = - 12.9 —
golden ges = 622.7 —	golden fis = 577.3 —
Scheme 21: plus Molecule v = + 45.4 —	ges = 622.7 —

b) Scheme 15 shows the exact change (to three decimals) in the middle figure (15 tones) and gisis. The number of cents diminish or increase evenly with

$$\left\{ \begin{array}{l} 5.741 \text{ cents in all horizontal Fifth-Axis} \\ 1.456 \text{ — — — oblique Third —} \end{array} \right\}$$

representing »the 2 building materials« in the syntonic mixing-system: Fifths and Thirds:

1) + 1.373 cents eis	2) — 2.912 gis	3) — 2.912 gis	4) 5 syntonic e = 378.2 feses +
— 1.456 plus e	+ 2.829 plus ♯	— 2.829 minus ♯	5 golden e = 356.8 feses
— 0.083 gisis	— 0.083 gisis	— 5.741 g	distance = 21.4 = 1 Comma

c) Rational explanation: Super division of the octaves e ... e' and as ... as' (Supplement-pairs) gives »retrograde gisis« and »direct feses« respectively.

d) Similar methods (but of inferior structure) were up to 1558 used in the three famous systems of chance by: (see Scheme 16);

	Zero-Axis	The unit	
		increases	decreases
(X unknown author) before 1511,	a c es	+ $\frac{1}{8}$ as	- $\frac{1}{8}$ e
The Bohemian, Arnold Schlick 1511 ..	as c e	+ $\frac{1}{4}$ a	- $\frac{1}{4}$ es
The Italian, Gioseffo Zarlino 1558	ccs c cis	+ $\frac{1}{7}$ { as a	- $\frac{1}{7}$ e - $\frac{1}{7}$ es
here, Scheme 14 1930	feses c gisis	+ $\frac{1}{16}$ as	- $\frac{1}{16}$ e

Scheme 16 shows the 3 Fifths oscillating about the golden g, »the truth«:

	X	Zarlino	Golden system	Schlick
the result g:	694.8	695.8	696.2	696.6
by means of:	$\frac{5}{15}$	$\frac{4}{14}$	$\frac{4}{15}$	$\frac{4}{16}$
in decimals:	= 0,333	= 0,286	= 0,267	= 0,250
Distance from the ideal:	+ 0,066	+ 0,019	Ideal	- 0,017

Addendum.

Scheme 17 shows diagrams of 7 Minors and 6 Majors, some of them in pairs symmetrically, see pages 10, 15 and 20.

Glareanus's erroneous nomenclature of the medieval scale-type should once for all be obliterated from the literature of music, and should be replaced by that recommended in Scheme 17, compare Helmholtz's heartfelt cry: »Übrigens werde ich Glareans Namen nicht brauchen es wäre überhaupt besser, wenn man sie vergessen möchte,« repeated by Ellis: »But I shall not use Glareanus's names It would be better to forget them altogether.« (Note 21).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

Scheme 19: tertiary distances as arcs in a circle, see pages 7 and 16.

Scheme 20: construction of golden tones by means of parallel lines, see page 24.

Scheme 21: Golden tones in cents with 4 decimals. see page 24.

Scheme 22: examples of logarithms as training in the use of my Constant K = 0.6005714, see page 7.

In Scheme 23 is set forth a proposal for a system of notation on music-paper, staff, in all other tone-systems than the 12-toned temperament, with the suggestion, offered by K. Steensen (Note 22) for a rational arrangement of the \sharp and \flat .

Scheme 24 shows how the cents can easily be calculated by taking the difference between the logarithms for Partial-tones, the numbers of which are respectively the numerator and the denominator in a tone-fraction, according to the table constructed by the organist Kai Kroman, Copenhagen, for example:

$$\text{oriental } d = \frac{10}{9} \left\{ \begin{array}{l} \text{Logarithm for Partial tone No. 10} = 3.9863 \\ \text{--- --- --- --- ---} \\ \text{difference ... } 182.4 \text{ cents} \end{array} \right.$$

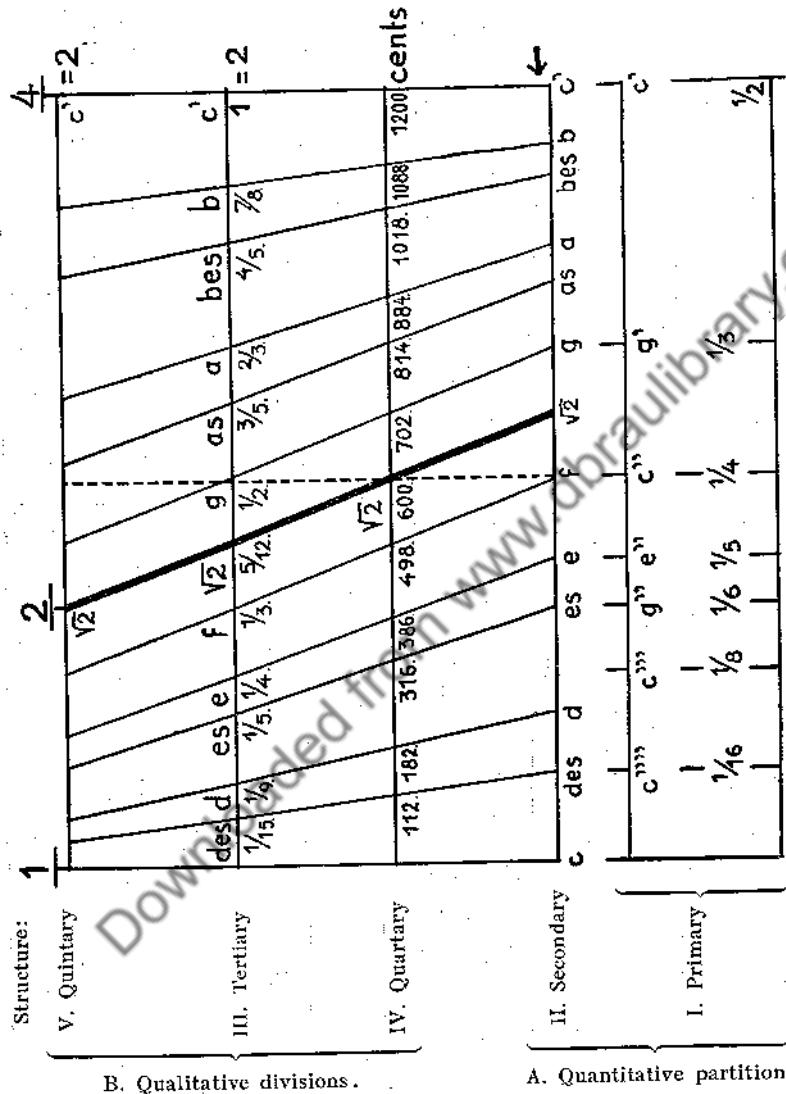
The word »Partial-tone« is preferred to »over-tone« in order to avoid the confusion, which Ellis characterises as »great confusion, Tyndall's **erroneous** translation« (Note 21) between

the German *ober* and the English *upper*: adjective
— *über* — — over: preposition, adverb.

Bibliography.

Page	Note	
4	1	Professor E. v. HORNBOSTEL: »Die Massnorm als kulturgeschichtliches Forschungsmittel«, »Festschrift P. W. Schmidt«, 1928, page 311 and »Analecta et Additamenta«; »Die Herkunft der alt peruanischen Gewichtsnorm«, pp. 235-258.
5	2	Ibid: »Musikalische Tonsysteme« in »Handb. der Physik«, Bd. VIII, Chp. 9, p. 438.
	3	Prof. E. v. HORNBOSTEL and ROBERT LACHMANN: »Das indische Tonsystem bei Bharata und sein Ursprung«, in »Zeitschrift für vergleichende Musikwissenschaft«, Berlin, 1933, p. 89, Note 1.
	4	Professor E. v. HORNBOSTEL, Berlin, above mentioned; »Musikalische Tonsysteme«, Chp. 9, p. 429 and Prof. E. v. HORNBOSTEL and R. LACHMANN: »Asiatische Parallelen zur Berbermusik«, in »Zeitschrift f. vergl. Musikw.«, 1933, p. 10.
7	5	Professor JOSEPH YASSER: »A Theory of Evolving Tonality«, New York, 1932, p. 21, Note.
10	6	Professor A. Z. IDELSOHN: »The Features of the Jewish Sacred Folk-Song in Eastern Europe« in »Acta Musicologica«, Leipzig IV, 1932, pp. 22-23.
11	7	Prof. HORNBOSTEL and LACHMANN: »Zeitschr. f. vergl. Musikw.«, 1933, p. 80; VICTOR MAHILLON: Catalogue descriptif du Musée instrumental du Conservatoire de Bruxelles, T. 1. 2. éd. Gand 1893, p. 93 ff.
15	8	HELMUT BITTER, Istanbul: »Der Reigen der Tanzenden Dervische«, in »Zeitschr. f. vergl. Musikw.«, 1933, p. 28-40 and appendices 5; and concerning Blas-quinte see: HORNBOSTEL: Note 3 in »Handbuch der Physik«, Bd. VIII, Chp. 9, p. 431.
17	9	Administrator F. LASSEN LANDORPH: »Javanese Gamelan« in »Annals for Music«, published by »Dansk Musikselskab«, Copenhagen 1923, pp. 7-23, const.
		Prof. v. HORNBOSTEL: »Handb. der Physik«, VIII, Chp. 9, p. 433.
19	10	Dr. phil. ALFRED JONQUIÈRE: »Grundriss der musikalischen Akustik«, Leipzig 1898, p. 120, and H. v. HELMHOLTZ: »Tonempfindungen«, 1913, p. 457.
20	11	DIRECTOR GODTFRED SKJERNE: Danish translation of Plutarchos's dialogue on music, with explanation, Copenhagen 1909, pp. 1-214; Prof. JOH. WOLF: »Handb. der Notationskunde«, 1913; and Professor GUIDO ADLER: »Handb. der Musikgeschichte«, Berlin 1924.
22	12	LUDW. SONNENBERG: »Der goldene Schnitt«, Progr. des Kgl. Gymnasiums zu Bonn, 1881.

Page	Note	
22	13	(Pioneer) LUCA PACIOLI: »Divina proportionē«, 1509, into German in »Quellschriften für Kunstgeschichte«, Wien 1889, pg. 196; JOHANNES KEPLER: Letter from Prague of 12th May 1608 to Professor JOACHIM TANK († 1609), in »Opera omnia«, C. FRISCH's edition, Vol. I, Frankfurt 1858, pp. 140, 145 r and 375—384. ALEXANDER BRAUN in »Nova acta Acad. Leop. Carol.« XV 1831, pp. 195—402; L. and A. BRAVAIS: »Lois géometriques des spirales« in »Annales des sciences naturelles«, 2. Serie, Paris 1837, Bot. Tom. 7, pp. 42—110; GABRIEL LAMÉ: »Comptes rendus de l'academie des sciences«, 1844, vol. III, Juillet—Décembre, pg. 867—870; Prof. H. E. TIMERDING: »Der goldne Schnitt«, 1918; Ing. VILII. MARSTRAND: »Arsenalet i Piræus og Oldtidens Byggeregler (rules of building of antiquity) Copenhagen 1922.
—	14	LUDWIG KAISER: »Über die Verhältniszahl des goldenen Schnitts«, Leipzig 1929, p. 122; THORVALD KORNERUP: »Die Hochteilung der Octave«, Copenhagen, Oct. 1930; the main contents is embodied in this treatise.
26	15	Physicist HANS CHRISTIAN ØRSTED: »Samlede og efterladte Skrifter«, Copenhagen; B. 7. 1858, pp. 93—95.
28	16	Director GODTFRED SKJERNE: above mentioned »Plutarchos«, 1909, p. 77.
—	17	Professor JOSÉ WÜRSCHMIDT, Tucuman in Argentine: »Die rationellen Tonsysteme im Quinten-Terzen Gewebe«, in Scheel's Zeitschrift für Physik, Berlin 1928, p. 526, (in the German text tone »b« is used for the tone $\frac{15}{8}$); Spanish translation: »Los sistemas de sonidos racionales«, Buenos Aires, 1928, p. 3.
—	18	Professor JOSEPH YASSER: »A Theory of Evolving Tonality«, New York 1932, pp. 278—84.
—	19	Professor LOUIS KELTERBORN: »Die Quinten Spirale«, Darmstadt 1929.
29	20	Professor T. N. THIELE's »number-figures«, grafic statements by means of points of systems in »Beretning om Naturforskermødet« in Copenhagen 1872.
32	21	HERMANN v. HELMHOLTZ's »Tonempfindungen«, 1913, p. 441 and Ellis translation, »Sensations of Tones«, 1912, p. 269, rep. p. 25, Note.
33	22	Arrangement of ♯ and ♭ according to K. STEENSEN: »Den musikalske Skrive-maade«, in Skjerne's periodical »Musik«, Copenhagen 1922, p. 46.



Scheme 1. Graphic sketch of 5 kinds of calculation (primary-quintary of tones).

17 Persian intervals Cents.	Doric-Sruti Nos.	17 Sruti Nos.				13 permanent tones Persian Doric Minor on 7 Tonics (key-notes).										Number of touches
		13 permanent Nos.		4 Persian blank, variable Sruti-Nos.												
		Cents	Cents	Cents												
90	17	1200			c	e	e	c	c	c	c	c	c	c	c	7
22	16	1110					b +									2
92	(15)		1088	b												—
90	14	996			bes —	bes —			hes —	hes —	bes —					5
22	13	906				a +	a +				a +	a +				4
92	(12)		884	a												—
90	11	792			as —			as —	as —							3
90	10	702			g	g	g		g		g	g	g	g	g	6
24	9	612													ges —	1
90	8	588							fis +							1
90	7	498			f	f	f	f	f	f	f	f				6
22	6	408				▲	e +				e +	e +				3
92	(5)		386	e												—
90	4	294			es —	es —		es —	es —							4
22	3	204				▲	d +	d +	d +	d +	d +	d +	d +	d +	d +	5
92	(2)		182	d												—
90	1	90			des —			des —	des —							2
—	0	0			c		c	(c)	(c)	(c)	(c)	(c)	(c)	(c)	(c)	0
1200	Total	13	+ 4	= 17											Total	49

Scheme 2, P. The 17-toned Persian Sruti-System; fis + = ges 2 —.

Number of touches	17 permanent tones					17 permanent Nos. 53 Comma-temperament.	Indian names,	5 Indian				
	Probable Indian	Lydian—Phryian Sa Grama Double Lydian Ma Grama on 5 Tonics (key-notes)						22 Sruti 53 Commas	Names	Cents		
		c	d +	f	g							
7	c	e	b +	c	c c	c	22	53	Sadja			
3			b	b			21	49	Nisada			
4	b				b		20	48				
3	bes—			bes—			18	44		(19) 45 bes 1018		
7	a +	a +	a +		a +	a +	17	40	Dhaiavata			
3	a			a			16	39				
1					as —		15	35		(14) 34 gis 772		
9	g	g		g	g	g	13	31	Pancama			
1					ges—		12	27				
4		fis +		fis +	fis +	fis +	11	26		(10) 25 fis 568		
5	f		f		f		9	22	Mahijama			
5		e +		e +	e +	e + e +	8	18	Gandhara			
4	e		e		e		7	17				
1			es—				5	13		(6) 14 es 316		
9	d +	d +	d +	d +	d + d +	d +	4	9	Rsabha			
1			d				3	8				
3		des—			des —		2	4		(1) 9 cis 70		
0	c		c	c	c		0	0	Sadja	— — — —		
70	Total.						Total: 17 + 5 = 22.					

Scheme 2, J. The 17-toned Sruti-System, extended to 22-toned Indian-system.

53-toned No.	4	5	9
Greek names	Iota ←	Theta	Eta
Pythagorean	des ---	des	d +
Cents.....	90	112	204
Particles about	683 ³ / ₄	675	640
5-partition 8 ³ / ₄	+ 35	= 43 ³ / ₄ .
53-toned No.	4	5	8
Arabian names	{ nim ← zirkula zirkula → tig zirkula
presumably	des ---	des	d
Cents.....	90	112	182
Particles	684	675	648
4-partition 9	+ 27	= 36

Scheme 3. Great and small retrograde Triads: Greek Triad....., 5 Commas,
and presumably Arabian Triad .., 4 — .

53 Temp. Nos.	g.	as —	a. a $\frac{1}{4}$	bes —	b. b +	c.							
Direct Triads.	31.	35.	39, 40.	44.	48, 49.	53.							
≡ 5 Commas....													
Retrograde Triads.													
Persian Nos.	10.	11.	12, 13.	14.	15, 16.	17.							
Indian Nos.	13.	15.	16, 17.	18,	20, 21.	22.							
53 Temp. Nos.	c.	des —	d. d +	es —	e. e +	f.							
Direct Triads	0.	4.	8, 9.	13.	17, 18.	22.							
≡ 5 Commas....													
Retrograde Triads.													
Persian Nos.	0.	1.	2, 3.	4.	5.	6.							
Indian Nos.	0.	2.	3, 4.	5,	7, 8.	9.							

Total 53 Commas.

Scheme 4. The 13 permanent oriental tones and 4 variable tones (d e a b). Total 17 Persian.

53-toned temperament = No. about	0	11	21	32	42	53	
Malayan translation.....			$\frac{1}{2}$ (nem)		(2 X tengah)		
Malayan name.....	bartung	goeloe	tengah	lima	{ anam nem }	barang	Scale
Vibration numbers	270	310	356	409	470	540	Lassen Landorph
Ellis cents	0	240	480	720	960	1200	Cents
Quintal 5-toned temp.	c	eses ('1/2 -)	f —	g +	ais ('1/2 +)	c'	Symmetrical
1 molecule (eses).....	240	240	240	240	240	240	= 1200
5 Pythagorean tones.							
53-toned temperament = No. about	0	7	20	26	31	35	49
Malayan translation.....	charming			exchanging	No. 5	No. 6*	charming
Malayan name.....	manis	goeloe	tengah	pelog	lima	{ anam nem }	manis
Vibration numbers	279	304	362	395	418	443	Material
Ellis cents	0	148	448	590	700	792	Lassen Landorph
Syntonic names	c	cisis	cis	{ ges 2 — fis + }	g	as — b + c'	Cents
(material)							

No.	1. c	2. cisis	3. eis	4. fis +	5. g.	6. as —	7. b +	8. c'	
Quartary intervals:									cents = 1200
Harmonic Tritonos, intervals ...	148	300	142	110	92	318	90		Total 590
nearly 4-division	cisis cisis (es -)	cisis	cisis	cisis	cisis	cisis	cisis	cisis	
Small Medium, Greek "Diazuxis decreased": $\frac{16}{15}$	1 + 2 + 1								— 110
Harmonic upper Tetrachord with the small Third in tune, intervals									
nearly 5-division									— 500
10—partition.....	c	cisis	eis	fis +					Material
Particles nearly	720	658	554	512					Particles
Spaces = 208 =		62 +	104 +	42					
— abbreviated		3 +	5 +	2					
5—partition.....									Material
Particles nearly...									Particles
Spaces = 120									
— abbreviated									

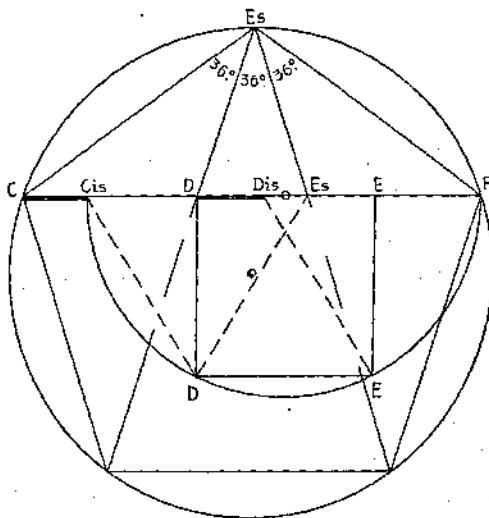
Scheme 5, continuation. Javanese Gamelan (salendro and pelog respectively) built up upon Lassen Landorff's vibration-numbers, in "Annals for Music", Copenhagen 1923, pp. 7—23.

Scheme 6. The 23 Pythagorean instrument-tones, of which 19 are song-tones from c to b₅, (from 8 Fourths to 10 Fifths).

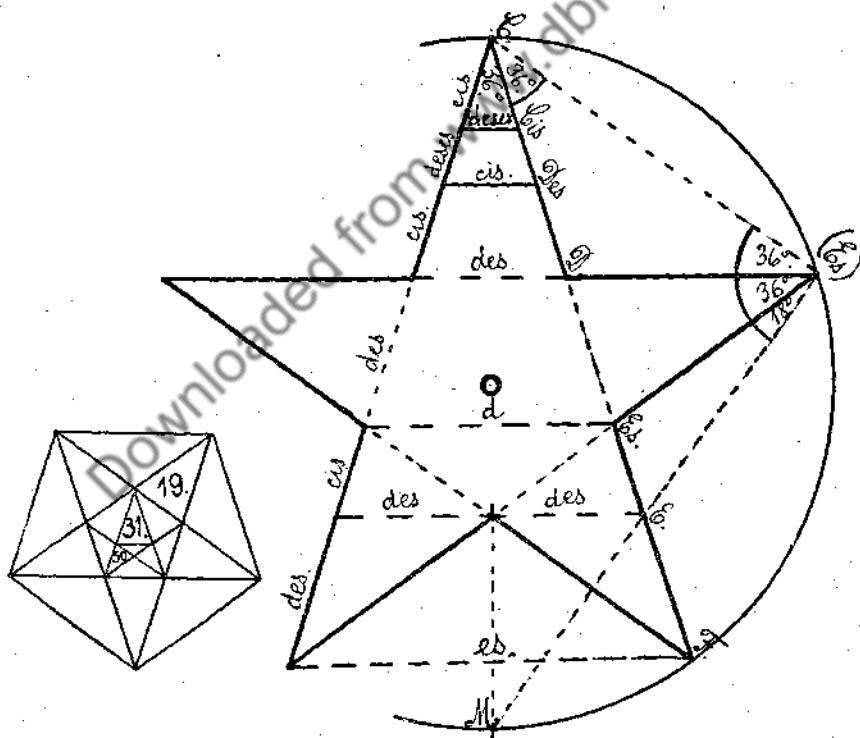
	Hyper.	Doric Minor.	Hypo.
	Minor triple chord: yx	Minor triple chord: yx	Diminished triple chord: yy
a+	c	e+	b+
Sub-Dominant.	Tonic.	Dominant.	
1	2 3	10 20	

Scheme 7. The tone-Nos for 7 instrument-tones are presumably founded on triple-chords (i. e. chords of Thirds, English triads). The symbol x signifies Great Third, v Small Third.

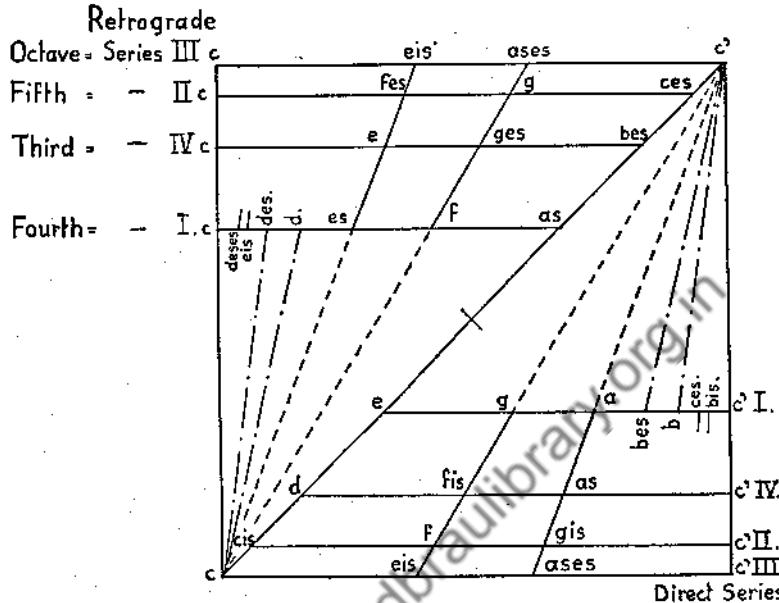




Scheme 8. Geometrical construction of great and small double super division (the golden cut), by means of a Pentagon and a square respectively.



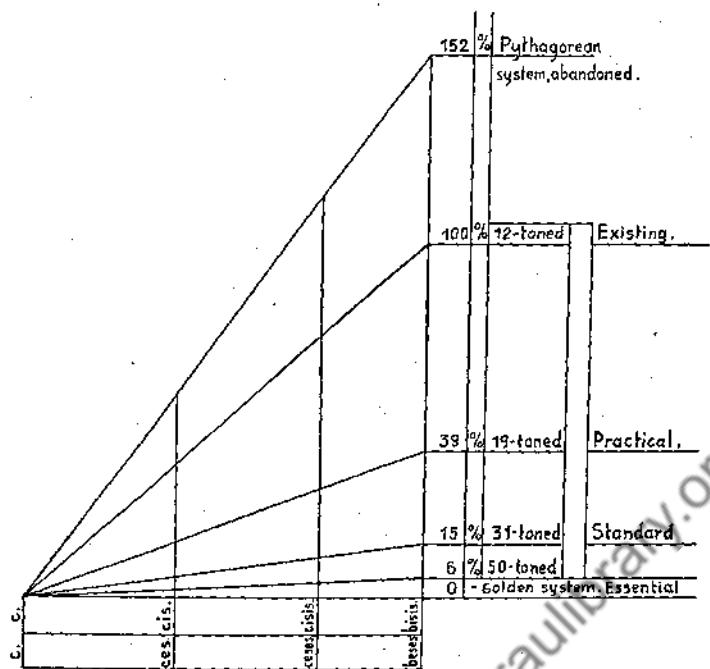
Scheme 9. The golden system constructed by means of the Pentagram.
The sides of the isosceles triangles are Molecules,
for examples deses and cis in 19-toned section.
basis and deses » 31 » » »



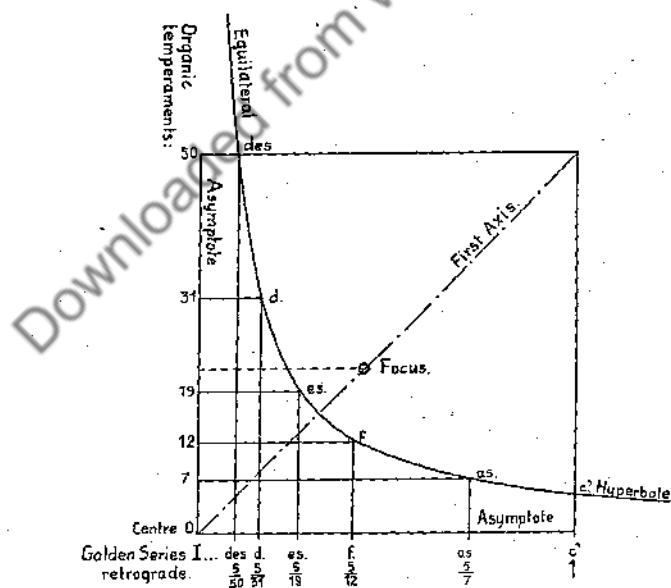
Scheme 10. Great double super division (golden cut) in 4 relations (Series).

A temperament too small	Organic temperament		Organic temperament		Inorganic temperament	
	12-tones	19-toned	31-toned	24-toned		
No. 1 { cis. 100	cis. No. 1	63.2	cis. Nr. 2	77.4	cis. No. 1	50
{ des. 100	des. 2	126.3	des. 3	116.1	des. 2	100
2 d. 200	d. 3	189.5	d. 5	193.5	cisis. 3	150
3 es. 300	es. 5	315.8	es. 8	309.7	d. No. 4	200
5 f. 500	f. 8	505.3	f. 13	503.2	es. 6	300
8 as. 800	as. 13	821.1	as. 21	812.9	f. 10	500
					as. 16	800

Scheme 11. Golden Series No. I used as a means of valuation for organic and inorganic temperaments. The lower c always No. 0.



Scheme 12. Graphic description of the deviation (falsity) of 4 organic temperaments and the Pythagorean system.



Scheme 13. An equilateral hyperbole through the tones having No. 5 in different organic temperaments; des d es f as, the golden Series I retrograde.

Stage №	Tones -2Commas	Tones minus 1Comma	Central-tones without plus or minus				Tones plus 1Comma	Tones +2Commas
2.	cisis- +19	gisis- +15	disis- +11				Zero Axis	
3.	bis- +12	fisis- +8	cisis- +4	gisis- 0	disis- -4			
4.	dis- +9	ais- +5	eis- +1	bis- -3	fisis- -7	cisis- -11	gisist- -15	disis- -19
5.	fis- +6	cis- +2	gis- -2	dis- -6				
6.	F- +19	C- +15	g- +11	d- +7	a- +3	e- -1	b- -5	
7.	Pythag. des- +20	as- +16	es- +12	bes- +8	F- +4	C- 0	G- -4	d+ -8
8.					des- +5	as- +1	es- -3	bes- -7
9.					f+ -8	-11	c+ -15	g+ -19
10.	b3b2- +19	feses- +15	ceses- +11	geses- +7	deses- +3	ases- -1	eses- -5	beses- -9
11.					b3b- -4	feses- 0	ceses- -4	geses- -8
12.						Zero Axis	deses- -12	b3b- -19
							fezes- -11	ceses- -15

Scheme 14. Change of the syntonic system into golden tones by means of numbers of k ($= \frac{1}{15}$ of a Comma), in 9 oblique figures, each containing 15 tones, formed on 9 central-points of plus 15, 0 and minus 15 k respectively.

Stage	Zero - Axis.				
3	gisis — 0.083				
4		ais. + 7.114	eis + 1.373	bis — 4.367	fisis + — 10.108
5		fis + 8.569	# = cis + 2.829	gis — 2.912	dis + — 8.652
6	d + 10.052	a + 4.285	e — 1.456	b — 7.196	
7		f + 5.741	c 0	g — 5.741	Pythagorean tones,

Scheme 15. The middle figure (and gisis) in Scheme 14, showing exact change into golden tuning.

0. 0. 0. Zero-Axis.				
The highest cents.	+ $\frac{1}{4}$ cis. 76.0			0 gis. 772.6
A. Schlick 1511.		0 e. 386.3		
$\frac{1}{4} = 5.377$ cents.	0 c. 0			
Authoritative golden system. k = $\frac{1}{15}$ = 1.434 cents.	+ $\frac{4}{15}$ cis. 147.6	+ $\frac{1}{15}$ eis. 458.4	0 gisis. 843.2	
			- $\frac{8}{15}$ gis. 769.7	
	0 c. 0	- $\frac{1}{15}$ e. 384.9	- $\frac{4}{15}$ g. 696.2	
The lower cents	0 cis. 141.3			- $\frac{2}{7}$ gisis. 837.2
G. Zarlino 1558.	0 cis. 70.7	- $\frac{1}{7}$ eis. 453.9	- $\frac{2}{7}$ gis. 766.5	
$\frac{1}{7} = 3.072$ cents.	0 c. 0	- $\frac{1}{7}$ e. 383.2	- $\frac{2}{7}$ g. 695.8	
The lowest cents	- $\frac{1}{3}$ cis. 63.5			- $\frac{2}{3}$ gis. 758.3
X. before 1558.	0 c. 0	- $\frac{1}{3}$ e. 379.1		
$\frac{1}{3} = 7.169$ cents.		0 es. 315.6	- $\frac{1}{3}$ g. 694.8	
			0 ges. 631.3	

Scheme 16. Three systems oscillating around the authoritative golden system, k = $\frac{1}{15}$ of a Comma.

1. Doric-Tritonos Minor. Tritonos == ges.

2. Doric Minor.

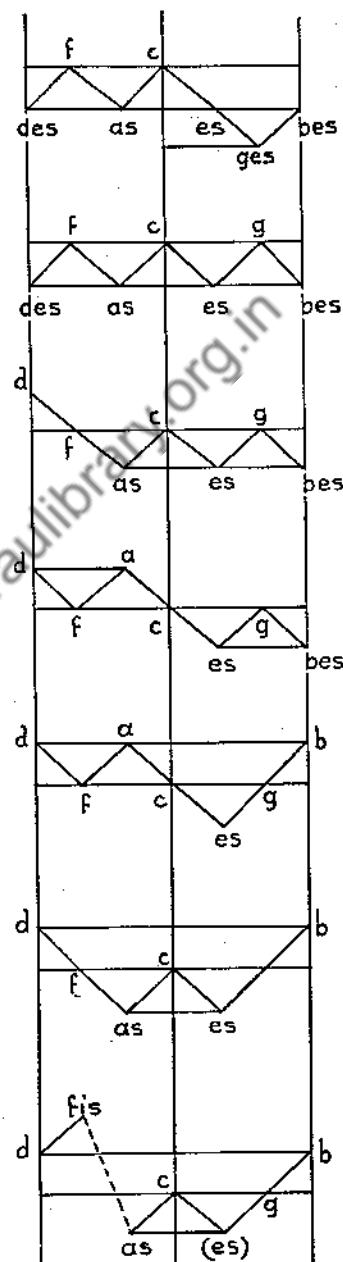
3. Phrygian-Doric (or Aiolian) Minor,
our descending melodic Minor.

4. Phrygian Minor, double symmetric.

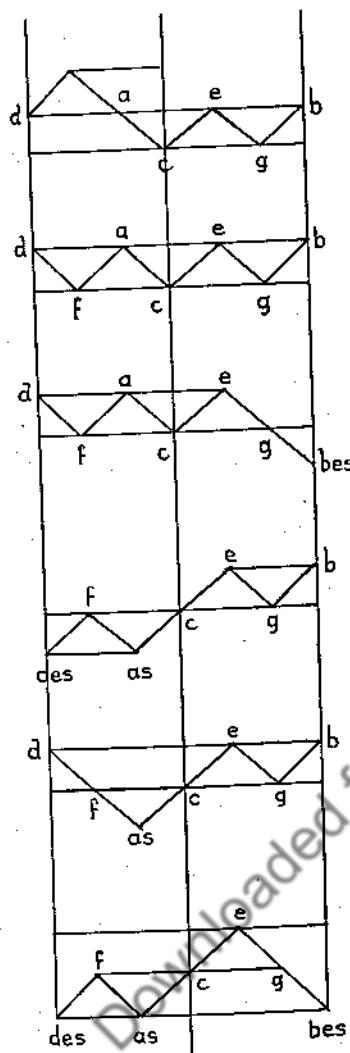
9. Phrygian-Lydian Minor,
our ascending melodic Minor.

11. Phrygian-harmonic Minor,
Ernst F. E. Richter 1853.

13. Turkish »uneven Tritonos ... uneven
harmonic« Minor, Neu-eser (Rauf
Yekta Bey).



Scheme 17.



7. Tritonos-Lydian Major. Tritonos = fis.

6. Lydian Major, Indian Ma.

5. Lydian-Phrygian (or Jonic, or Jastic) Major,
Indian Sa.

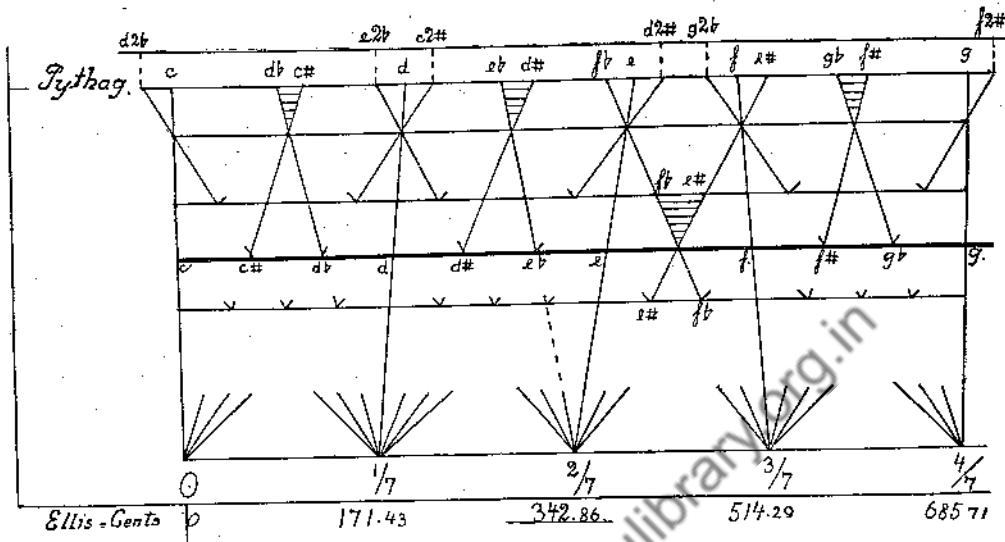
8. Double harmonic Major, Hedjaz-Kar (A. Z.
Idelsohn).

10.a. Lydian-harmonic Major, Rimsky Korssakoff 1893.

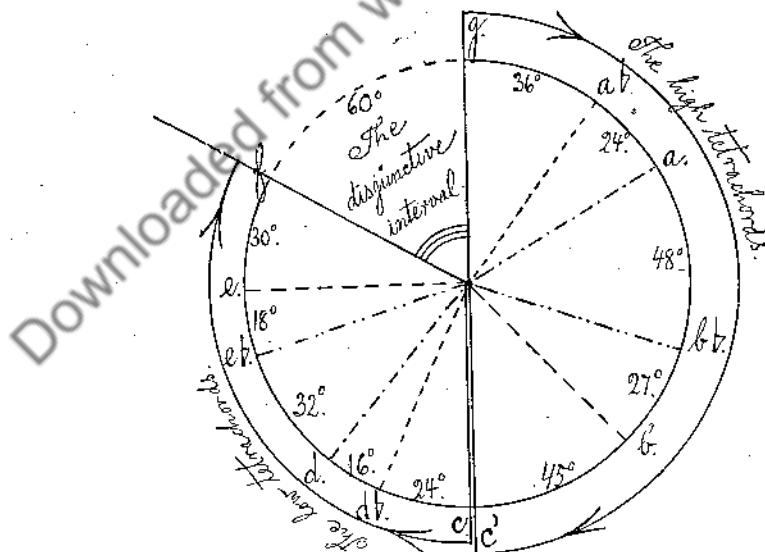
10.b. »Lydian-uneven harmonica«, Turkish Suzanak, (Rauf Yekta Bey).

12. Harmonic-Doric Major, Ahavah Rabbah,
(A. Z. Idelsohn).

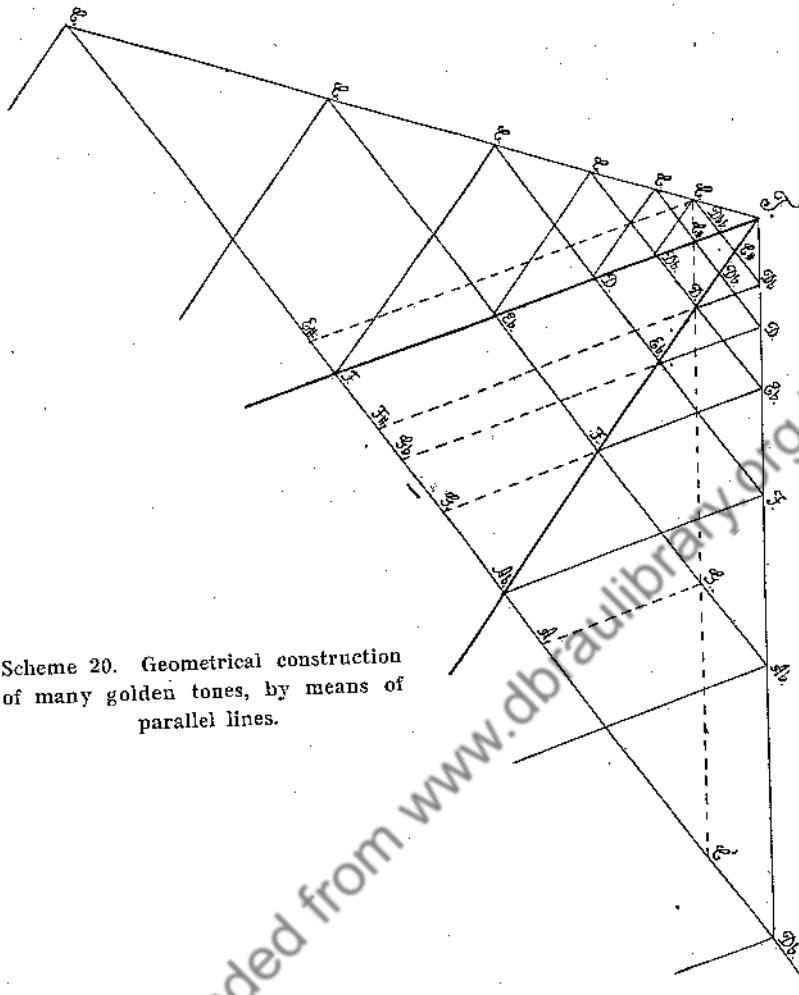
Scheme 17. Names and diagrams of 13 scale-types, on the Tonic c, in syntonic
or golden tuning on c; 7 Minors and 6 Majors; No. 1—7 Pythagorean.



Scheme 18. Lines through corresponding tones in the Pythag. system and 5 temperaments.



Scheme 19. Two Tetrachords tertiarily graduated as a semi-circle and a sector.



Scheme 20. Geometrical construction
of many golden tones, by means of
parallel lines.

No.	Cents	Dec.	No.	Cents	Dec.	No.	Cents	Dec.
26	16	bes 1007	5711	28	17	b 1081	0724	30
21	13	as 815	1421	23	14	a 888	6434	18 bis 1154
16	10	ges 622	7132	18	11	g 696	2145	15 als 962
11	7	fes 430	2842	13	8	f 503	7855	12 gis 769
8	5	es 311	3566	10	6	e 384	8579	9 fis 577
3	2	des 118	9276	5	3	d 192	4289	7 eis 458
29	18	ces 1126	4987	31	19	c 0	—	4 dis 265

or No. 0 Two 19-toned Molecules. { $t = cis$
 $v = deses 45$ } 4263

Scheme 21. The 19-toned golden section, with 19-toned and 31-toned Nos.

Logarithmic Examples:

1) The decimal fraction of the golden Fifth?

$$\begin{array}{rcl} \text{Log. } 0.696.2175 (\text{g}) = & \dots & 0.842.7431 - 1 \\ \text{minus Constant K..} & = & \text{minus } 0.600.5714 \end{array}$$

$$\begin{array}{rcl} \text{Log. difference} \dots & = & 0.242.1717 - 1 \\ \text{to which corresponds} & \approx & 0.174.6512 \dots \end{array}$$

to which again corresponds the decimal
fraction 1,495.34

Syntonic g = $\frac{8}{5}$ = 1,5 is a little larger.

How many particles?

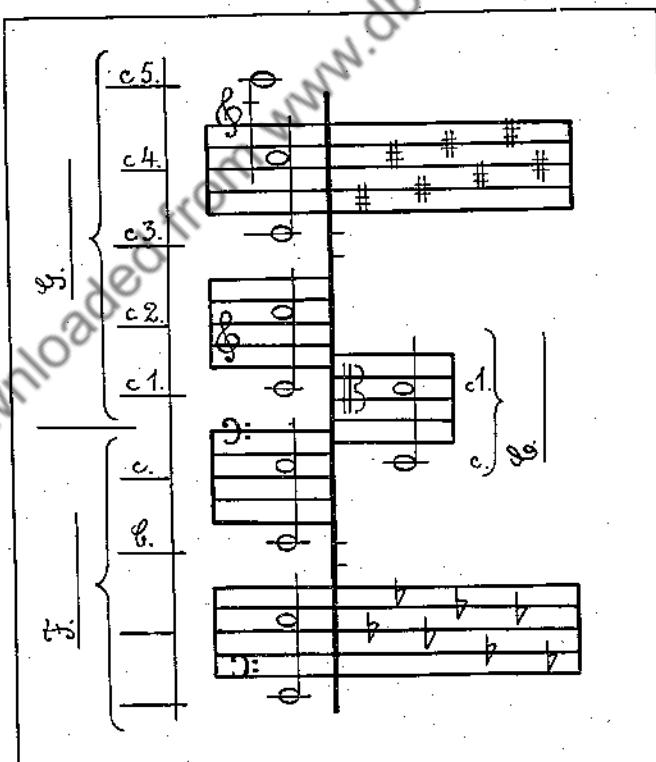
$$\begin{array}{rcl} \text{Log. } 720 \text{ particles} \dots & \dots & 2.857.3323 \\ \text{minus the above} \approx \dots & & 0.174.6512 \dots \end{array}$$

$$\text{Difference} \dots 2.682.6813$$

to which corresponds $\approx \dots$ 481,594

For syntonic g* 480 particles a little less.

Scheme 22, compare Chapter I, B, 4.



Scheme 23. B new 5-lined staff (1931) applicable to other temperaments than the 12-toned temperament.

The first 5 octaves			The 6th octave			The 7th octave					
	Cents			Cents		c — g	Cents	g — c'	Cents		
			$2^5 = 32$	6 000 0	$2^6 = 64$	c 7	200 0	96 g 7	902 0		
C 1	0 0			3 053 3		5	26 8	7	19 9		
c. $2^1 = 2$	1 200 0			4 105 0		6	53 3	8	37 7		
g 3	1 802 0			5 155 1		7	279 3	9	55 2		
c. $2^2 = 4$	2 100 0		d + 6	6 203 9		8	305 0	100	72 6		
e..... 5	2 786 3			7 251 3		9	30 2	1	989 9		
g	6 102 0			8 297 5		70	55 1	2	006 9		
7 3 368 8				9 342 5		1	379 7	3	23 8		
c. $2^3 = 8$	4 600 0		e. 40	6 386 3		2	403 9	4	40 5		
d + .. 9	3 803 9			1 429 1		3	27 8	5	57 1		
e..... 10	3 986 3			2 470 8		4	51 3	6	73 5		
11 4 151 3				3 511 5		5	74 6	7	089 8		
g..... 12 4 302 0				4 551 3		6	497 5	8	105 9		
13 4 440 5		f# + 5	6 590 2		7	520 1	9	21 8			
14 4 568 8			6 628 3		8	42 5	110	37 6			
b .. 15 4 688 3			7 665 5		9	64 5	1	53 3			
c. $2^4 = 16$	4 800 0		g. 8	6 702 0		80 e	586 3	2	68 8		
17 4 905 0			9 737 7		1	607 8	3	84 2			
d + .. 18 5 003 9			50 772 6		2	29 1	4	199 5			
19 6 997 5			1 806 9		3	50 0	5	214 6			
e..... 20 5 186 3			2 840 5		4	70 8	6	29 6			
1 270 8			3 873 5		5	691 3	7	44 4			
2 351 3			4 905 9		6	711 5	8	59 2			
3 428 3			5 937 6		7	31 5	9	73 8			
g..... 4 5 502 0			6 968 8		8	51 3	120 b 8	288 3			
5 572 6			7 999 5		9	70 9	1	302 6			
6 640 5			8 029 6		90	7 790 2	2	16 9			
7 705 9			9 059 2		1	809 4	3	31 0			
8 768 8		b. 60	7 088 3		2	28 3	4	45 0			
9 829 6			1 116 9		3	47 0	5	58 9			
b ... 30 5 888 3			2 145 0		4	65 5	6	72 7			
31 945 0			3 172 7		5	883 8	7	386 4			
c. $2^5 = 32$	6 000 0		$2^6 = 64$	7 200 0		96 g 7	902 0	128 c' 8	400 0		

Scheme 24. Special cents-table, showing 6 octaves of the Partial-tones 1—128, built up upon the original table by K. Kroman, Organist, Copenhagen.

Example: b, Partial-tone No. 15 30 60 120 ..
cents 4.6883 = 5.8883 = 7.0883 = 8.2883

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Addendum.

- Names and Diagrams of 13 scale-types in syntonic or golden tuning.
 Lines through corresponding tones in 5 temperaments.
 Tetrachords tertiary graduated as a semi-circle and a sector.
 Geometrical construction of many tones, by means of parallel lines.
 The 19-toned golden section, with 19-toned and 31-toned Nos.
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 - b) Secondary partition of a string, particles (spaces).
 - c) Relativity of interval-distances.
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